Ground Plan Geometries in Suger's St-Denis

A Prototype for Altenberg

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The Cistercian church at Altenberg, to which Norbert Nußbaum has devoted a great deal of attention in recent years, should be understood in several different senses as a descendant of St-Denis. First, of course, it draws on the legacy of St-Denis in a very broad sense, because Suger's chevet is widely seen as the first Gothic monument worthy of the name. Second, Altenberg features seven radiating chapels clustered around a hemicycle bay whose geometrical center lies slightly to the east of the straight choir bays. In this respect, Altenberg belongs to a tradition that can be traced from St-Denis to the choirs of Amiens, Beauvais, and Cologne Cathedrals, and to the abbey churches of Longpont and Royaumont, to cite just a few of the more prominent examples.[1] As Nußbaum has noted, moreover, the chevets of St-Denis and Altenberg also share a striking feature seen in none of these other buildings: their piers are aligned radially so that a priest standing at the high altar would have a clear and unencumbered view of the windows in all seven radiating chapels.[2] (Ill. 1) This recurrence of this highly unusual feature strongly suggests that the designer of the Altenberg choir may have had St-Denis in mind when he was developing his ground plan, even though more than a century had elapsed between the dedication of Suger's chevet in 1144 and the beginning of work at Altenberg in 1259.[3]

Detailed geometrical analysis of the chevet plans of Altenberg and St-Denis reveals that the links between the two designs are even closer than has previously been imagined. Over the past several years, it has been my privilege to work closely with Norbert Nußbaum in exploring the logic of the Altenberg design. The results of our collaboration cannot be rehearsed at length in the context of this short article, but several brief points deserve note.[4] In Gothic design, the rotation of polygons

1 Norbert Nußbaum and Sabine Lepsky: Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Bd. 1 Die Choranlage, Bergisch Gladbach 2005, pp. 33–47. For a more political reading of the links between Altenberg and St-Denis, see Michael Davis, The Choir of the Abbey of Altenberg: Cistercian Simplicity and Aristocratic Iconography, in: Studies in Cistercian Art and Architecture, Volume Two, Meredith Parsons Lillich (ed.), Kalamazoo 1984, pp. 130–160, esp. 145–148.

2 Nussbaum/Lepsky 2005 (note 1), pp. 46–47, and Tafel 1. For discussion of this »crown of light« at St-Denis, see Sumner McKnight Crosby, The Royal Abbey of Saint-Denis from Its Beginnings to the Death of Suger (edited and completed by Pamela Blum), New Haven 1987, pp. 237–238.

3 For a variety of recent perspectives on Altenberg, see Norbert Nußbaum and Sabine Lepsky (eds.): 1259: Altenberg und die Baukultur im 13. Jahrhundert, Regensburg 2010.

4 Our full analysis will appear in Norbert Nußbaum and Sabine Lepsky: Gotische Konstruktion und Baupraxis an der Zisterzienserkirche Altenberg, Band 2, forthcoming.

was a frequently used proportioning strategy. Square rotation, or quadrature, is the best-known example of this process, but the idea can be applied equally readily to hexagons, octagons, and other such figures.[5] In the choir of Cologne Cathedral, for example, all of the radiating chapels fit into perfect 30° slices of a dodecagon, and all of the radii in the chevet can be related to each other by combinations of quadrature and dodecagon rotation.[6] The same principles can be shown to operate at Altenberg, although the 12-fold symmetry is not expressed explicitly because of the complex reorientation of the piers towards the perspectival viewpoint at the high altar. A crucial dimension in the establishment of the Altenberg plan is thus the 15.46 m radius from the chevet center to the corner of the large buttresses dividing the chevet from the straight choir bays. These buttresses, together with the smaller buttresses of the axial chapel, appear on archaeological grounds to have been the first elements built above the sockel.[7] It seems likely, then, that the layout of the Altenberg choir begun with the establishment of a large triangle defined by these buttresses, and inscribed within the semicircle whose diameter of 30.92 m coincides with the baseline of the chevet. Very similar geometrical logic seems to have governed the layout of the St-Denis chevet, whose scale closely matches that of Altenberg, as illustration 1 shows.

Since St-Denis has such singular prominence in the literature on Gothic architecture, and since Suger's comments about the importance of arithmetical and geometrical instruments in its layout are so frequently cited, it is somewhat surprising that no very satisfying explanation of its ground plan has previously been published.[8] Several different irregularities, however, complicate geometrical analysis of the design. All seven radiating chapels are articulated similarly, but their depths vary, with the axial chapel being the deepest, its neighbors intermediate, and the four others shallower. The salient buttresses sticking out between the chapels are not aligned with the radii of the chevet. Even in the supposedly straight bays, the columns do not align on simple east-west axes, and the baseline of the chevet appears slightly bowed, since the west columns of the hemicycle stand slightly west of the columns that flank them in the aisles. Sumner Crosby, who devoted much of his career to the exploration of St-Denis, argued in 1966 that these irregularities resulted at least in part from the use of multiple centers of curvature for the layout of the different sets of chapels, a layout that he compared to the epicylic geometry of Ptolemaic cosmology.[9] Peter Kidson, writing two decades later, called Crosby's article »the only serious attempt to come to grips« with the St-Denis plan, but he largely dismissed Crosby's conclusions, arguing that the greater depth of the axial chapel may simply have reflected liturgical

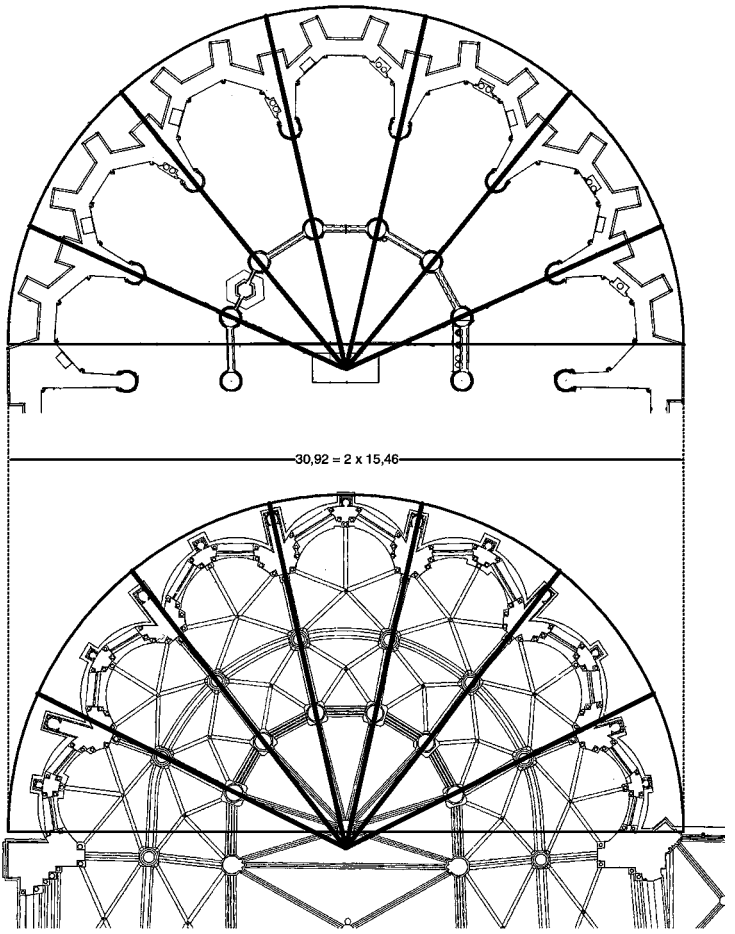
5 For a broadly ranging exploration of these themes, see Robert Bork, The Geometry of Creation: Architectural Drawing and the Dynamics of Gothic Design, Farnham 2011.

6 Bork 2011 (note 5), pp. 97–100.

7 Nussbaum/Lepsky 2005 (note 1), Tafel 2, p. 146.

8 Suger: De Consacratione, IV, line 15. Cited in: Erwin Panofsky (ed. and trans.): Abbot Suger on the Abbey Church of St-Denis and its Art Treasures, Princeton 1946, pp. 100–101.

9 Sumner Crosby: Crypt and Choir Plans at Saint Denis, Gesta 5 (1966), pp. 4–8. Interestingly, he alludes only very briefly to these cosmological ideas in Crosby 1987 (note 2), p. 241.

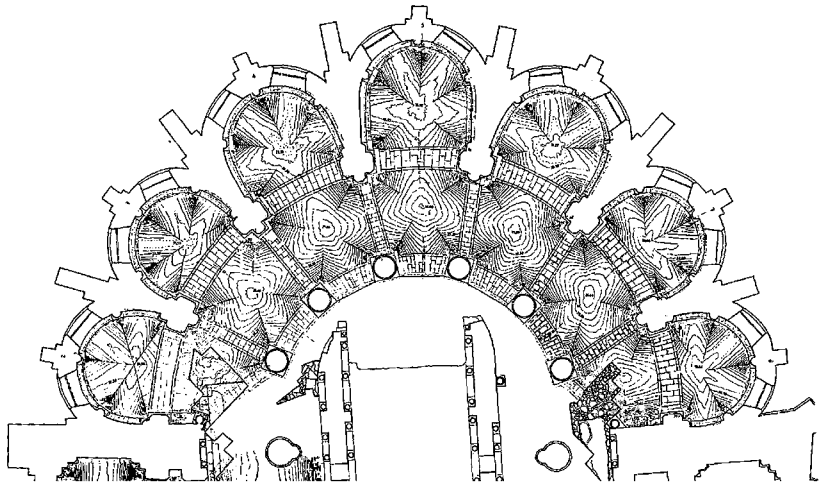


1 Comparative ground plans of chevets at Altenberg (above) and St-Denis (below), set to the same scale, with radii through column centers locating points of optical convergence

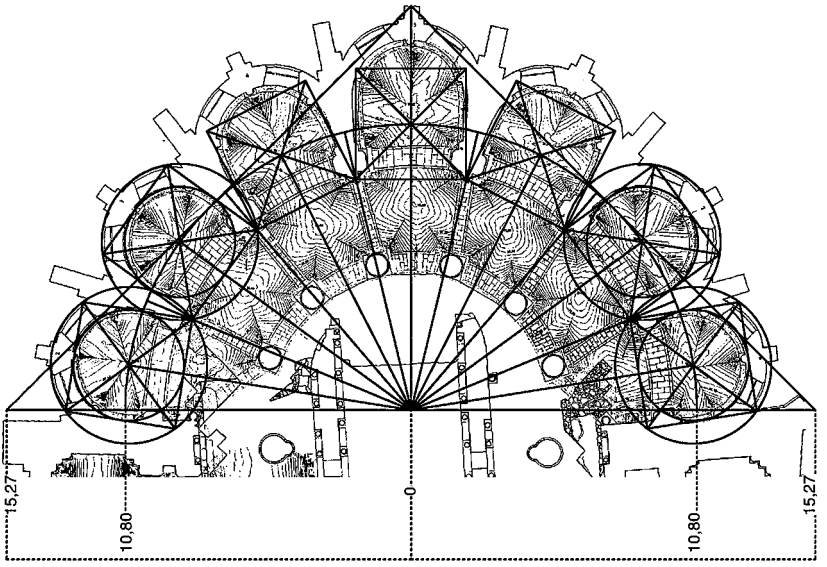
considerations, with the adjacent chapels taking on intermediate depths to make this distinction visually unobtrusive.[10] Kidson thus rejected both Crosby's epicyclic geometry and the associated cosmological interpretation. And, while Crosby had observed that the angle between the respective chapels was almost exactly 27°, Kidson argued that this quantity was an approximation to the 27.69° that one finds in a regular 13-sided polygon. This approximation, Kidson believed, may have reflected the builder's knowledge of an ancient formula, attributed to Heron of Alexandria, for approximating the side lengths of regular polygons inscribed within circles.[11] Kidson never goes on to explain, though, how the rest of the plan could have been developed, even if this formula was used for the overall subdivision of the chevet. Neither Crosby

10 Peter Kidson, Panofsky, Suger, and St Denis: Journal of the Warburg and Courtauld Institutes, Vol. 50 (1987), pp. 1–17, esp. 11.

11 Kidson (note 10), p. 15.



a Schematic plan with photogram of vaults



b Schematic plan with addition showing basic geometrical construction of chapels as proposed by Gould, and overall alignment with framing buttresses

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nor Kidson, therefore, provides a really adequate explanation for the overall formal and geometrical logic of the St-Denis chevet.[12]

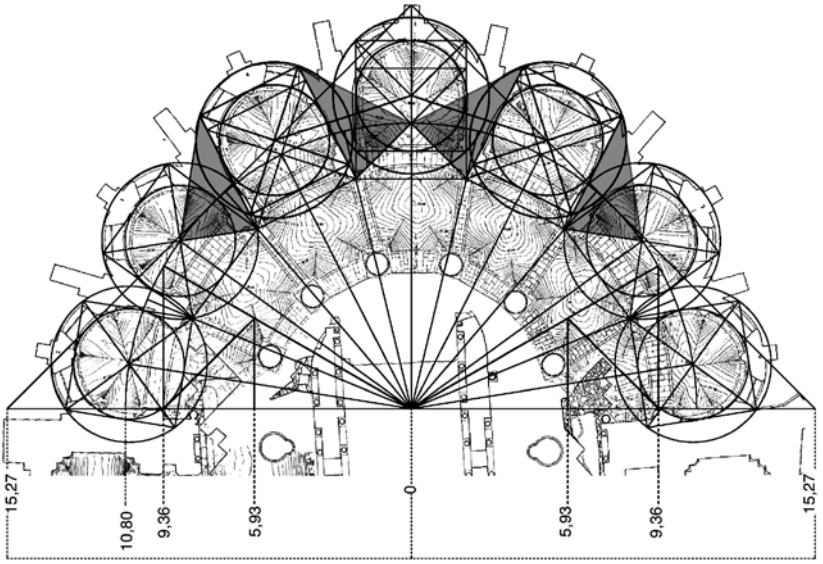
12 The more recent survey of the St-Denis chevet by Stefaan Van Liefferinge, similarly, concerns the arc-based layout of the ambulatory columns, rather than the logic of the design as a whole. See his essay Art, Architecture, and Science: Considerations on the Plan of the Chevet of Saint-Denis, in: Robert Bork, William W. Clark, and Abby McGehee (eds.): New Approaches to Medieval Architecture, Farnham 2011, pp. 147–157.

A far more compelling understanding of the St-Denis design can be gleaned by combining lessons drawn from the study of Altenberg with insights originally developed in the 1970s by Crosby's former assistant Richard Nash Gould. Although Gould's work was never formally published, it deserves to be taken very seriously, since it combines elegant simplicity with explanatory power.[13] Since Crosby had commissioned photogrammetric studies of the vaults at St-Denis, Gould had access to highly precise geometrical information about the shape of the crypt. (Ill. 2a) Taking advantage of an early computer system, Gould was able to unambiguously locate the geometrical centers and radii of all seven chapels, and of the chevet composition as a whole. He could thus verify the 27° angle between the chapel centers, which had already been identified by Crosby. Gould's crucial insight was to realize that this angle can be constructed by subtracting 45° from the 72° angle seen the geometry of pentagons. Equivalently, one can express 27 as 3/8 of 72, and 45 as 5/8 of 72, a formulation that introduces the relationship $72 = 8 \times 9$. This, in turn, relates to the idea that the 360° of a circle can be subdivided into 5 slices of 72°, or 40 slices of 9° each.

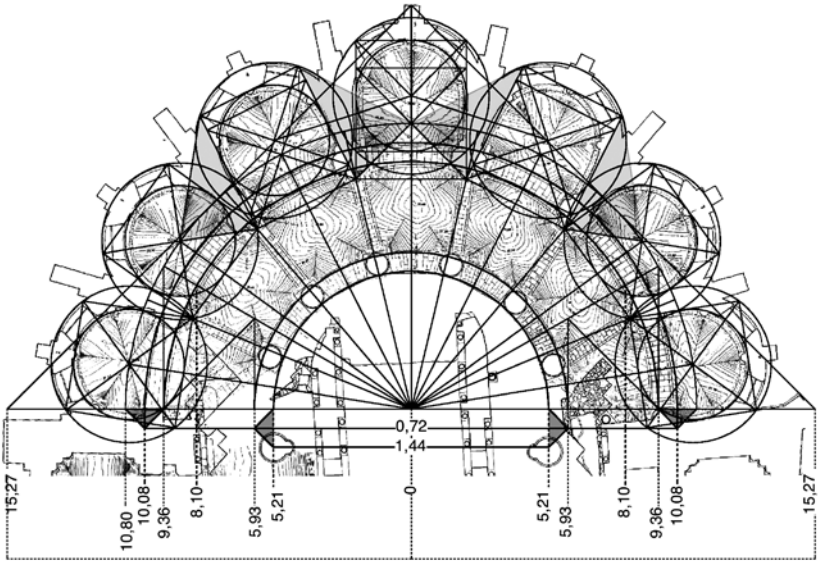
Gould's analysis of the St-Denis crypt begins with the establishment of a single geometrical center, and the subdivision of the chevet into such 9° slices. The centers of the chapels lie on lines that are displaced from the western baseline of the chevet by 9° for the first pair, by 9° + 27° = 36° for the second pair, by 36° + 27° = 63° for the third pair, and by 90° for the axial chapel. The radii from the chevet center to the centers of the first and second chapel pairs measure 10.80m; the three chapels nearest the axis are somewhat deeper, for reasons to be explained shortly, but one can begin their construction with centers lying on the same circle. The geometrical frames of the chapels can be understood as squares centered on these points, aligned with the radii to the chapel centers, and scaled such that their inner corners touch, as shown in illustration 2b. The sides of the squares measure 4.18 m. For the first and second chapel pairs, the internal wall surfaces are described by circles inscribed within these squares, which thus have radii of 2.09 m, while the external wall surfaces are described by circles circumscribed around them, which have radii of 2.96 m. The match between this simple geometrical construction and the building is impressively precise.[14] Before going on to discuss the more complex geometry of the three eastern chapels and other details of the crypt, it is worth noting that a diagonal tangent to the semicircle through the chapel centers intersects the baseline of the chevet composition at a point 15.27 m out from the chevet center. On the north, this intersection point is close to, but not quite coincident with, the northeast corner of the main buttress. On the south, the diagonal aligns with a spur on the buttress. These relationships recall the

13 Richard Nash Gould: The Crypt Plan at Saint Denis, 1974. I am grateful to William W. Clark for bringing this work to my attention, and to Richard Nash Gould for allowing me to cite his findings in this study. Gould's work is briefly mentioned in note 100 of Crosby 1987 (note 2), p. 241. It will also be published, at last, in an upcoming volume of studies on St-Denis that will be edited by William W. Clark, with Robert Bork and Andrew Tallon.

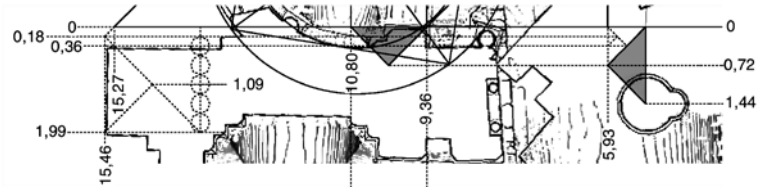
14 Gould 1974 (note 13) notes that this scheme locates these first four chapel centers to within 2 cm, with similarly good matches to the wall surfaces.



a Schematic plan showing layout of middle chapels as proposed by Gould, and introduction of 30° construction



b Schematic plan showing location of arches and columns



c detail of northern buttress, showing dimensions and proportions

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pattern seen at Altenberg, where the buttress corners 15.46 m from the chevet center coincide precisely with the baseline of the chevet composition. The kinship between these two designs will be considered at greater length later in this essay.

The shaded triangles in illustration 3a show how Gould proposed to explain the depth of the three chapels nearest the building axis. Each of these triangles has a single acute angle of 36°, and two wider base angles of 72°, so that each has proportions like the tip of a perfect five-pointed star. Each of their short bases is 2.96 m long, and aligned with the diagonal from the center of one of the chapel squares to its corners. Their longer sides each measure 4.79 m. Within the frames defined by the pairs of shaded triangles, therefore, the chapel-bounding squares 4.18 on a side can be pushed back away from the chevet center by 4.79 m – 4.18 m = 0.61 m. This displacement accurately describes the increased depth of the third pair of chapels, compared to the first and second pairs closer to the chevet baseline. From the centers of this third pair of chapels, finally, lines can be struck slightly eastward at 72° to the building axis, converging on the center of the axial chapel, which is thus 1.02 m deeper than the »regular« first and second pair. Again, Gould's scheme matches the fabric of the building with truly impressive precision, although it still leaves many aspects of the St-Denis chevet design unexplained.[15]

To round out this picture of the St-Denis ground plan, it is helpful to consider lessons drawn from the study of Altenberg's Cistercian church. There, as in the closely related Cologne Cathedral, subdivision of the chevet into 30° slices plays an important role in the composition. Application of this scheme to the St-Denis crypt quickly yields important results, as illustration 3a begins to show. Lines drawn at 30° to the chevet baseline intersect the arc of radius 10.80 m passing through the chapel centers at points 9.36m from the building centerline, since 9.36 = 10.80 × cosine (30°).[16] A semicircle of this new radius describes the inner face of the thick arch separating the crypt ambulatory from the chapels. And, as illustration 3b shows, the outer face of the arch is 10.08 m from the chevet center, or exactly halfway between the two semicircles just described, since (10.80 + 9.36)/2 = 10.08. This arch is thus 0.72 m thick, where 10.80 – 10.08 = 0.72. Meanwhile, a diagonal struck in from the end of the 9.36 m arc will intersect the previously described 30° line at a point 5.93 m from

15 Gould 1974 (note 13) notes that this scheme locates the center of the axial chapel to within 6 m, and the centers of the two adjacent chapels to within 1 cm.

16 This use of the 30° regulating line can also be inferred at Notre-Dame in Paris. In his presentation »Divining Proportions in the Information Age,« given at the conference »Proportional Systems in the History of Architecture« in Leiden in March, 2011, Andrew Tallon described the results of a rigorous laser scan of the Notre-Dame choir, revealing that the radius of the main hemicycle was 6.65 m measured to the pier centers, with each of the two successive ambulatories adding 5.76 m steps outward. Although he did not consider the ratio between these numbers, it should be noted that 5.76/6.65 = .866 = cosine (30°). The width of the aisles, in other words, can simply be found by inscribing a hexagon within the circle framed by the arcade axes. The facets of this hexagon 5.76 m from the building centerline will align closely with the inner faces of the arcade plinths. With this information in hand, the rest of the Notre-Dame choir geometry can be generated fairly readily, as I plan to demonstrate in an upcoming article.

the building centerline. This dimension serves as the radius of the semicircle describing the outer face of the thick arch beneath the hemicycle. This arch, like the outer one, has a thickness of 0.72 m, so its inner face is 5.21 m from the chevet center. All six of the hemicycle piers stand on this inner arch, but only the northern one is well aligned with the relevant buttress. As one moves further to the south, these round piers become further and further displaced, but the interval between their centers remains nearly constant at some 2.80 m.[17] This strongly suggests that their location was determined by counting off this regular module around the previously established circle on which their centers lie. A very similar construction was evidently used to set the locations of the hemicycle piers at Altenberg, but in the German building the counting seems to have started from the east rather than the north, so that the troublesome asymmetry seen in the St-Denis crypt was avoided.[18] The two piers at the base of the hemicycle stand 1.44 m, or two arch thicknesses, west of the chevet's geometrical baseline, on axes separated from the building centerline by the same 5.21 m interval that defines the inner face of the hemicycle arch.[19]

Illustration 3c shows a detail of the north buttress on the chevet baseline, which deserves particularly close consideration because of its relationship both to Altenberg and to the geometry of the upper chevet story at St-Denis itself. The northern face of the buttress stands almost exactly the same distance from the building centerline that the analogous buttress face does in Altenberg, namely 15.46 m. In Altenberg this same dimension is the radius of the great semicircle governing the overall chevet composition, and the corner of the buttress coincides perfectly with one corner of this semicircle. At St-Denis, though, this master radius is 15.27 m, or $\sqrt{2}$ times the 10.80 m radius through the crypt chapel centers, which means that the buttress face is some .18 m further to the north than it »should be« for this relationship to pertain. It is significant in this context, moreover, that the east face of the buttress is some .36 m west of the chevet baseline, while the flange on its southern edge, next to the chapel, stands some .18 m west of the baseline. It is tempting to interpret these intervals as one half and one quarter, respectively, of the .72 m arch thickness already seen in the crypt. If the buttress face had been established one of these .18 m intervals further to the east, then its corner would have aligned with the great diagonal heading northwest from the axial chapel, as may well have been intended.[20] If this had been

17 Crosby cites a distance of 2.70 m between the centers, but this seems low based on the photogram. See Crosby 1987 (note 2), p. 241. Clark identifies this dimension as 2.07 m, but this appears to be a transposition of digits. See William W. Clark: Suger's Church at Saint-Denis, in: Paula Lieber Gerson (ed.): Abbot Suger and Saint-Denis, New York 1986, p. 110.

18 Nussbaum/Lepsky 2005 (note 1), p. 57.

19 These piers in their present form date to the thirteenth century, but they must have the same locations as their twelfth-century predecessors, since they lie directly beneath columns that serve to support Suger's twelfth-century ambulatory vaults.

20 It is even conceivable that this displacement may have resulted from cutting back the surface of the original twelfth-century masonry, as was evidently done in various parts of the building by the nineteenth-century »restorer« François Debret. See Sumner Crosby, The Abbey of Saint-Denis 475–1122, New Haven 1942, p. 9.

the case, then the overall depth of the buttress would have equaled ten of these small .18 m intervals, as the small dotted circles in illustration 3c indicate, and its centerline would have stood .109 m west of the chevet baseline. This may all sound objectionably hypothetical, but consideration of the upper story of the St-Denis chevet shows that the center of the buttress, as defined by its articulation on the interior, actually was aligned with this notional centerline.

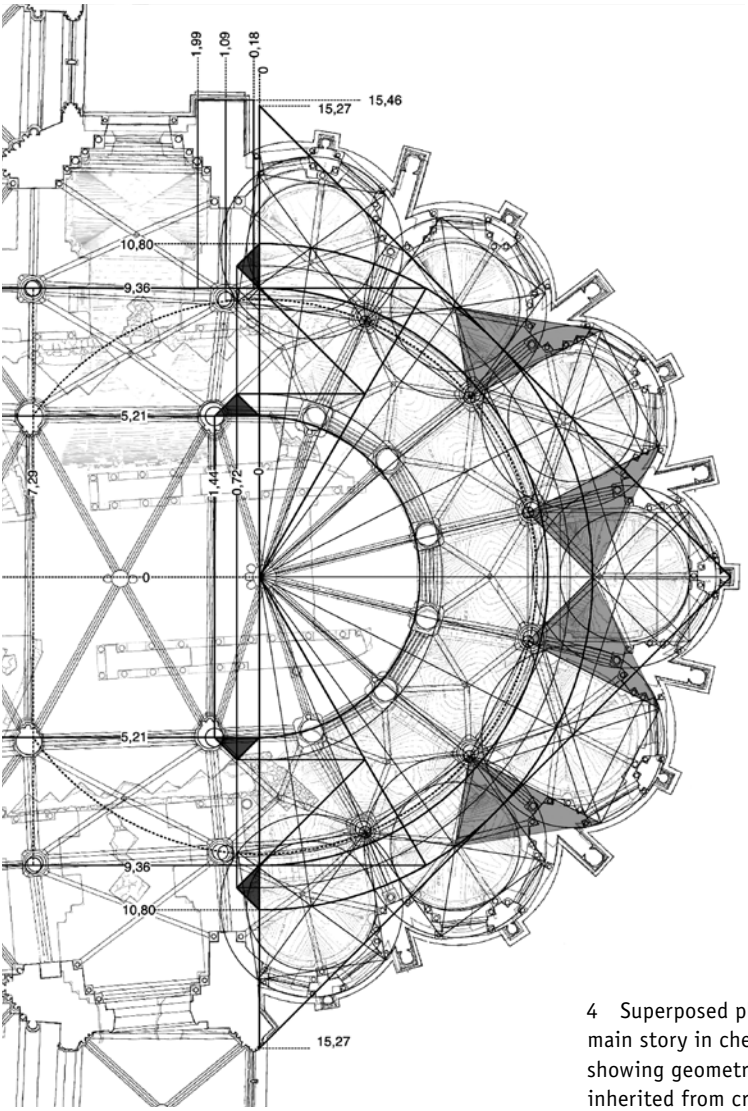
Illustration 4 begins to show how many of the key points established in the layout of the St-Denis crypt would go on to influence the design of the main chevet story immediately above. The pier centers at the hemicycle base in the crypt, for example, locate the centers of the analogous piers at the eastern edge of the last choir straight bay, 5.21 m out from the building centerline and 1.44 m west of the chevet baseline.[21] As Crosby already realized, the free-standing ambulatory columns are centered immediately above the front edges of the walls separating the crypt chapels, which is to say that these columns are centered 8.96 m from the chevet center, right above the points where the square corners touch in illustration 2b.[22] When a circle through these centerpoints is swept to the west, it intersects the arcade axes at points 7.29 m west of the chevet baseline, thus defining the length of the first straight bay. The ambulatory piers at this bay division, meanwhile, stand 9.36 m out from the building centerline, so that they align with the inner face of the thick chapel-entrance arch in the crypt. Because of the discrepancy between this dimension and the 8.96 m span seen at the base of the hemicycle, the inner aisle bays taper slightly from west to east. All of these alignments are more precise on the northern side of the building, which appears on archaeological grounds to have been constructed before its southern pendant.[23] On the north side, the previously discussed buttress at the base of the chevet is perfectly aligned on a north-south axis, and its visual center as defined by the two colonnettes on its face coincides closely with the axis 1.09 m west of the chevet baseline. The transverse rib emerging from this buttress continues that north-south axis over to the free-standing ambulatory column, before kinking westward to meet the arcade column 1.44 m west of the chevet baseline. On the south side of the building, the analogous columns have essentially the same places, but the transverse arch in the outer aisle has an inclination like that of the inner one, which may help to explain why the east face of the southern buttress was built at an angle to the chevet baseline. This angling, which causes the buttress to intrude awkwardly into the space of the southernmost chapel, must have been established already in the layout of the crypt, but it is tempting to imagine that imprecise copying of the northern components must have had something to do with it.

Illustration 5 adds a few more details to this picture, showing how the layout of the St-Denis choir anticipated the scenographic arrangement seen at Altenberg. The

21 As already observed in note 19, the current piers at the corners of the first straight bay all date from the thirteenth century, at both the crypt and upper levels, but their locations must respect those of their twelfth-century predecessors.

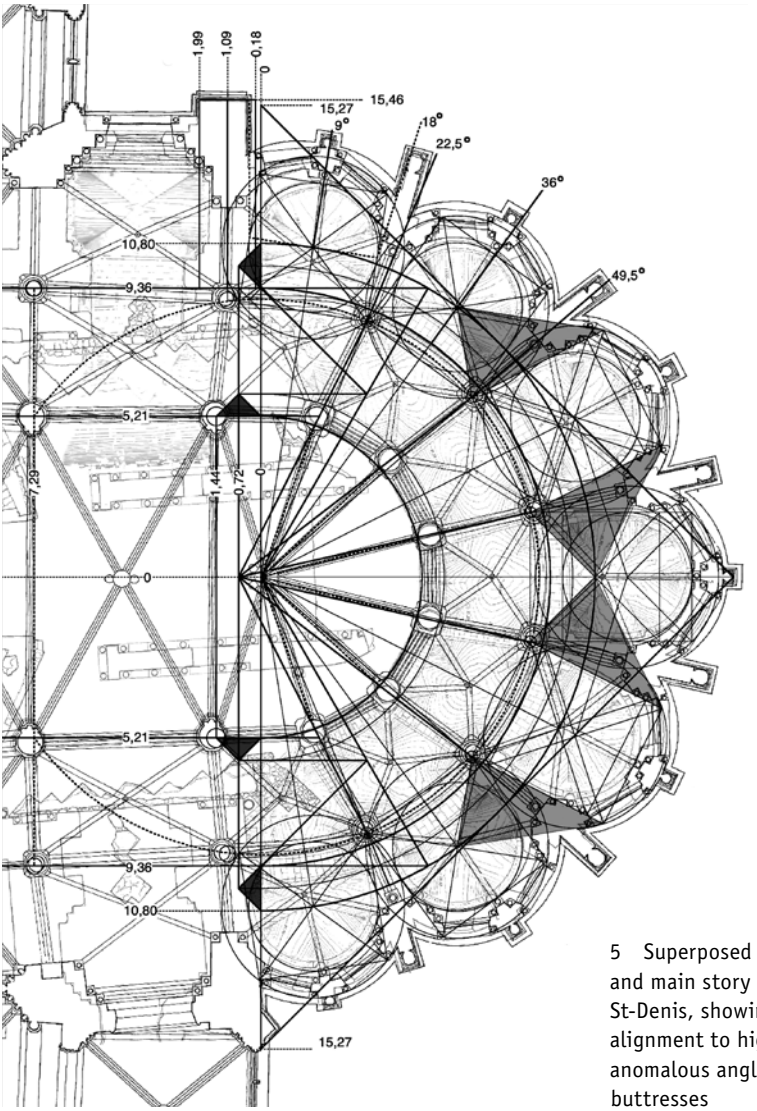
22 This superposition of stories is seen in Crosby 1987 (note 2), p. 238–239.

23 Clark 1986 (note 17), p. 111.



4 Superposed plans of crypt and main story in chevet of St-Denis, showing geometrical elements inherited from crypt design

optical center of the St-Denis chevet sits some .72 m, or one arch thickness, to the west of the geometrical center. Radial lines struck from the outer ambulatory columns to this center locate the transverse ribs of the inner ambulatory. The slender colonnettes on the inner faces of the hemicycle piers stand where these radial lines cut the circle of radius 5.21 m that is framed by the arcade axes of the straight bays. The ribs of the thirteenth-century hemicycle vault converge to the geometrical center of the chevet, and the same was probably true of the vault built in Suger's day, as well. At St-Denis, as at Altenberg, the use of an optical center separate from the geometrical center introduces kinks into the radial elements connecting the hemicycle center to



5 Superposed plans of crypt and main story in chevet of St-Denis, showing optical alignment to high altar and anomalous angles of northern buttresses

the outer buttresses. At Altenberg, these elements include flying buttresses, which make awkward angles with their pier buttresses.[24] At St-Denis – which may or may not have had flying buttresses in its twelfth-century state – the problem becomes even more acute, because the outer buttresses themselves are at seemingly odd angles to the chapels they frame.[25]

24 Nussbaum/Lepsky 2005 (note 1), 47.

25 On the problem of flying buttresses at Suger's St-Denis, see David Stanley: The Original Buttressing of Abbot Suger's Chevet at the Abbey of Saint-Denis, in Journal of the Society of Architectural

One possible rationale for these strange buttress angles may well involved notions of symmetry, as interpreted by a viewer looking at each successive chapel from the outside. The center of the northernmost chapel, as described previously, lies on a line offset by 9° from the chevet baseline; this relationship can be seen near the top of illustration 5. The large buttress to its west, meanwhile, is aligned on a perfect north-south axis. Naïve considerations of local symmetry would then suggest that the buttress separating the first chapel from the second one should have a slope of 18° with respect to the baseline, which in fact it does, as the dotted line in the illustration shows. This local perspective ignores the more global geometry of the chevet, whereby the buttress subdividing these chapels really »should« lie on a true radius to the geometrical center of the chevet, with a slope of 22.5°, where 22.5° is halfway between the 9° and 36° radii to the chapel centers. This first buttress is therefore twisted by roughly 22.5° – 18° = 4.5°, and it is also displaced too far to the west, so that it flanks the second window of the chapel just as the large buttress at the chevet baseline flanks the first window. This displacement has kind of cascade effect, which is clearly visible in the second chapel. The small subsidiary buttress in the second chapel should in theory lie on a radius 36° from the chevet baseline, but it is pulled measurably to the west, so that it will fall halfway between the radically displaced buttress to its west, and the more plausibly located one to its east, between the second and third chapels. Although this buttress aligns more closely with its major radius than the first one had, it is still shifted very slightly to the west, and its slope is slightly greater than the 49.5° that it should have. So, the buttresses flanking the first chapel pinch too tightly, and those flanking the second chapel pinch too loosely, but each chapel composition would look locally symmetrical to a viewer from the outside. Further to the east and to the south, the pattern becomes more complicated, not only because the three eastern chapels are deeper than the others, but also because errors seem to have accumulated as construction moved to the south, as evidenced both by the placement of the crypt hemicycle piers, and by the misalignment of the large southern buttress separating the chevet from the straight bays. It seems clear, nevertheless, that considerations of local symmetry help to explain the otherwise rather perplexing anomalies in the buttress layout at St-Denis.[26]

The later builders who so radically transformed Suger's chevet from the thirteenth century onwards obviously had to pay attention to its geometrical structure, including especially its buttress alignments. It is striking in this context that the small spur buttress added to the eastern face of the buttress by the southernmost radiating chapel has a diagonal face that aligns perfectly with the great diagonal heading towards the east end of the axial chapel. The corner of this spur buttress, in fact, sits 15.27 m south of the chevet center, so that it coincides with the corner of the great rotated

Historians, Vol. 65, No. 3 (Sep., 2006), pp. 334–355; and the critical responses by Robert Mark and Andrew Tallon in Journal of the Society of Architectural Historians Vol. 66, No. 1 (March, 2007), pp. 136–139.

26 It is hard to tell if Crosby was identifying a similar principle in the rather ambiguously phrased passage about the buttress alignments in Crosby 1987 (note 2), pp. 240–241.

square that circumscribes the circle of radius 10.80 m struck through the centers of the crypt chapels. This alignment is particularly interesting because the back of the axial chapel could not have been seen directly from ground level, once the southern chapels had been constructed. This implies that the designer of the spur buttress really understood the geometrical logic of the twelfth-century design, and that he was not just sighting to convenient points while adding to the structure.[27]

Further evidence for such continuity of geometrical knowledge comes from consideration of the relationship between St-Denis and Altenberg. Altenberg, after all, shares with St-Denis not only the seven-chapel layout, but also the unusual optical alignment of its chevet piers, and its overall scale matches that of St-Denis to an uncanny degree. It can hardly be coincidental that both buildings have principal buttress faces almost exactly 15.46 m from their centerlines, for example. Both designs seem to have been generated from very similar figures, involving circles framed by rotated squares between their main buttress corners. Both designs, moreover, involve proportions based on 30° subdivisions of these circles, even though neither expresses this 30° subdivision as clearly and explicitly as the chevet plan of Cologne Cathedral. In view of all the similarities, it is tempting to imagine that the designer of the Altenberg choir may have visited and even measured St-Denis, during the very years when Suger's building was being transformed in the mid-thirteenth century. Perhaps the Altenberg designer even spoke with someone in the St-Denis workshop about the technique of aligning the baseline buttresses with the axial chapel, before deciding to build these components first at the German church.[28] Such narrative details must,

27 Unfortunately, the dating for the conception of this spur remains unclear. Crosby, in the color-coded foldouts from the first volume of his study, shows the spur as part of the Sugerian twelfth-century masonry at crypt level, but he shows its upper portion as part of the nineteenth-century rebuilding of the old sacristy, also known as the chapel of Saint Louis. See Crosby 1942 (note 20), endpapers. It seems implausible on stylistic grounds that the spur would have been conceived already in the twelfth century, when buttresses typically had simple rectangular outlines at ground level. Moreover, the color-coding of this crypt plan cannot be seen as reliable, since it also gives a twelfth-century date for the substructures of the thirteenth-century north transept. No such twelfth-century substructures existed that far out from the building, as Crosby himself demonstrated in the second volume of his study. See Crosby 1987 (note 2), pp. 270–271. Crosby also says that the masonry of the spur-bearing buttress extends for a meter »before it is replaced by the construction of the sacristy built in the fifteenth century.« See Crosby 1987 (note 2), p. 249. – Caroline Bruzelius, on the contrary, dates the chapel to the early fourteenth century; she does not discuss it at length, since her work emphasizes the main period of reconstruction between 1231 and 1281. See Caroline Bruzelius: The 13th-Century Church at St-Denis, New Haven 1985, p. 16. – Elizabeth Brown adds a bit more detail, claiming that this structure was originally built between 1299 and 1303, as a chapel in honor of the newly canonized Saint Louis, but she does not treat its architecture at length, either. See Elizabeth Brown: Saint-Denis: La Basilique, Orleans, 2001, p. 58. The spur is already visible on the anonymous eighteenth-century plan reproduced as illustration 98a in Crosby 1987 (note 2), p. 224, so it cannot be dismissed as the invention of nineteenth-century restorers. Putting all of this evidence together, it appears most likely that the twelfth-century southern buttress originally had a simple rectangular form like its northern pendant. The spur was probably introduced around 1299, by an architect who understood something of the geometrical logic of his twelfth-century predecessor, which he acknowledged with a characteristically diagonal late Rayonnant form.

28 Since Altenberg was begun already in 1259, its designer cannot have been inspired by the construction of the Saint-Louis chapel in 1299, however.

of course, remain hypothetical. The exchange of scaled drawings could have achieved many of the same effects. It is absolutely clear, however, that the design of Suger's St-Denis was a major influence on the design of Altenberg.

These results are of interest not only because of what they reveal about the continuity of Gothic design practices over wide temporal and geographical expanses, but also because of what they reveal about the logic of Gothic design itself. At its most fundamental level, Gothic design was inherently geometrical, involving constructions like polygon rotation that could be effected with the compass and straightedge. By applying these techniques recursively, medieval draftsmen were able to develop plans of great subtlety and complexity.[29] As the sophistication of the St-Denis plan demonstrates, these techniques were already well established by the middle of the twelfth century.[30] It is worth noting that all of the dimensions in the preceding discussion of the plan interlock, so that the scales of all of the parts are set with respect to each other. Since the generative process was geometrical, some of these relative proportions involved factors such as $\sqrt{2}$ and $\sqrt{3}$, quantities that are called irrational numbers because they are not expressible as ratios of whole numbers. To translate these schemes to the full-scale building site, however, scaled measuring rods were doubtless helpful. Suger's comment about the use of both arithmetical and geometrical instruments underscores this point, which receives further confirmation from metrological analysis of Gothic buildings, including Norbert Nußbaum's analysis of Altenberg. Studies such as these have the potential to reveal a great deal about the logic of Gothic design, but much work remains to be done in this field, as the relative neglect of the St-Denis choir plan indicates. It is hoped that this short article, by providing a new perspective on the design of that seminal building, will help to inspire more work on the geometrical logic of Gothic architecture.

Photo credits

1 top: source image is survey drawing of Altenberg Cistercian church by A. Steinmetz, 1910–11, modified by the author to remove extraneous labels. – 1 bottom, 4, and 5: source image for chevet plan is plate 1 from Crosby 1987 (note 2). – 2, 3, 4, and 5; source image is a combination of figures 110c (showing crypt vault photogram) and plate 2 (showing crypt structure outlines) from Crosby 1987 (note 2).

All added lines and dimensional labels are graphics by the author.

29 Bork 2011 (note 5).

30 This point deserves emphasis, because Robert Branner had argued in an influential article that drawing-based design practices did not begin to flourish in Gothic workshops until the thirteenth century. See Robert Branner, Villard de Honnecourt: Reims, and the Origin of Gothic Architectural Drawing, Gazette des Beaux Arts 61 (March 1963), pp. 129–146. The refinement of the St-Denis plan, though, could hardly have been achieved without the use of scale drawings. Geometrically precise building plans may even have been used a full century earlier, as my analyses of Notre-Dame at Jumieges have begun to suggest. Those results will be published as part of a large monograph on Jumieges edited by James Morganstern.