INTRODUCTION
Geometry and the Gothic Design Process

Explaining the Roots of the Gothic Aesthetic

Gothic church architecture powerfully conveys impressions of movement and restless growth. In many Gothic interiors, slender columns seem to soar heavenward, as if unconstrained by gravity, before sprouting ribs that knit together into complex vaults. Exterior elements such as spires and pinnacles further emphasize the sense of upward thrust, while leafy crockets and other foliate carvings literalize the organic growth metaphor. Gothic churches also have a crystalline quality that adds a dynamism of its own to the architecture. Both the buildings themselves and their small components generally have polygonal plans, with faceted surfaces meeting at sharp edges. The alignments between these surfaces define planes of reflection and axes of rotation about which the architecture seems to organize itself. The formal kinship between large and small-scale elements, meanwhile, recalls that seen in mineral crystals.[1] These features of Gothic architecture together suggest that the church building is a living microcosm of a divinely created cosmos.

The organicity and complexity of Gothic churches, which contribute so strongly to their aesthetic effect, have, ironically, obscured the working methods of the designers who conceived them. Because Gothic buildings tend to dazzle their beholders, their visual pyrotechnics can appear to defy rational analysis. The striking fact remains, however, that Gothic churches were created not by miraculous cosmic forces, but by preindustrial workers armed only with simple tools. The church designers, in particular, developed their plans using mainly the compass and the straightedge rule. These men would have received their early training in the stoneyard, but by 1200 or so, the greatest masters had begun to function principally as draftsmen, working out their designs in elaborate drawings before the start of each building campaign.[2] Fortunately, several hundred of these drawings survive to document the designers' creative practice in remarkably intimate detail. These drawings include compass prick holes, uninked construction lines, and other telltale traces of the draftsman's labor. Their proportions, moreover, directly reflect the designer's original vision, uncorrupted by the small errors and misalignments that can creep into full-scale buildings during construction. Careful analysis of these drawings, therefore, can reveal a great deal about the logic of the Gothic design process.

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[1] Self-similarity of this sort is also seen in the mathematical objects known as fractals, which began to be studied only after computers made it possible to examine the forms that result from the repeated application of simple rules to make complex structures. For an early study of these forms, see Benoit Mandelbrot, *Fractals: Form, Chance and Dimension* (San Francisco, 1977).

[2] A pioneering study of this pivotal phase is Robert Branner, "Villard de Honnecourt, Reims and the Origin of Gothic Architectural Drawing," *Gazette des Beaux-Arts*, 6th ser., 61 (March 1963): 129–46.

This book is called *The Geometry of Creation* because it seeks to explain the geometrical design methods by which Gothic draftsmen conceived their audacious building plans. By presenting a series of case studies of major Gothic drawings, it will demonstrate, in detailed step-by-step fashion, how simple geometrical operations could be combined to produce designs of daunting quasi-organic complexity. Ultimately, it will show that Gothic architecture was governed by procedural conventions, rather than by fixed canons of proportion. The Gothic tradition, in other words, treated the finished building as the physical trace of a dynamic design process whose internal logic mattered more than the shape of the final product. The seeming organicity of Gothic architecture, therefore, is more than skin deep. It reflects the basic character of Gothic architectural order, which differs fundamentally from the more static modular order seen in most subsequent western architecture.

To understand why a book like the present one can provide a valuable new perspective on Gothic creativity, it helps to briefly review the history of previous attempts to explain Gothic design practice. Gothic designers themselves, unfortunately, left behind no very satisfying treatises on the subject. This is hardly surprising, since their training emphasized visual rather than textual communication. Until the fourteenth century, at least, most Gothic designers would have been functionally illiterate, and ill-prepared to record their methods in writing.[3] The so-called portfolio of Villard de Honnecourt from the early thirteenth century admittedly includes many architectural drawings, geometrical figures, and associated commentaries, some of which will be considered in detail below, but a variety of factors suggest that it should be treated as something less than a fully reliable guide to then-current architectural practice.

The drawings in Villard's portfolio, which rank among the oldest extant Gothic drawings, differ in several respects from the genuine workshop drawings that survive from roughly 1250 and later. To begin with, some of them provide dramatic quasi-perspectival spatial cues that suggest a subjective point of view. In his large exterior view of the Laon Cathedral tower, for example, the tops of the tabernacles are pitched diagonally to suggest a view from below (Figure 0.1 top). These tabernacles, moreover, appear much larger than they do in the actual tower. Even in Villard's drawings that present information more accurately and objectively, like his plan for the Laon tower, his rough draftsmanship sets his work apart from later and more geometrically precise building plans drawn with a compass and rule (Figure 0.1 bottom). In yet other cases, Villard's drawings differ quite markedly from the buildings they purport to represent, sometimes in ways that suggest genuine misunderstandings rather than simply alternative design proposals; his drawings of the Reims Cathedral choir, for example, differ from the real building in terms of both proportion and detailing.[4] For these reasons, among others, modern scholars tend to believe that Villard de Honnecourt was not actually an architect, but rather an

[3] This is not to say, of course, that they lacked recognized expertise and high social standing, which was respected even by Scholastic writers. See Paul Binski, "'Working by Words alone': The Architect, Scholasticism and Rhetoric in Thirteenth-century France," in Mary Carruthers (ed.), *Rhetoric beyond Words: Delight and Persuasion in the Arts of the Middle Ages* (Cambridge, 2010), pp. 14–51.

[4] Quasi-perspectival cues are especially evident in Villard's views of the Reims chapels, folios 30v and 31r. The Reims choir buttressing is shown rather inaccurately on folio 32v. See William W. Clark, "Reims Cathedral in the Portfolio of Villard de Honnecourt," in Marie-Thérèse Zenner (ed.), *Villard's Legacy: Studies in Medieval Technology, Science and Art in Memory of Jean Gimpel* (Aldershot, 2004).

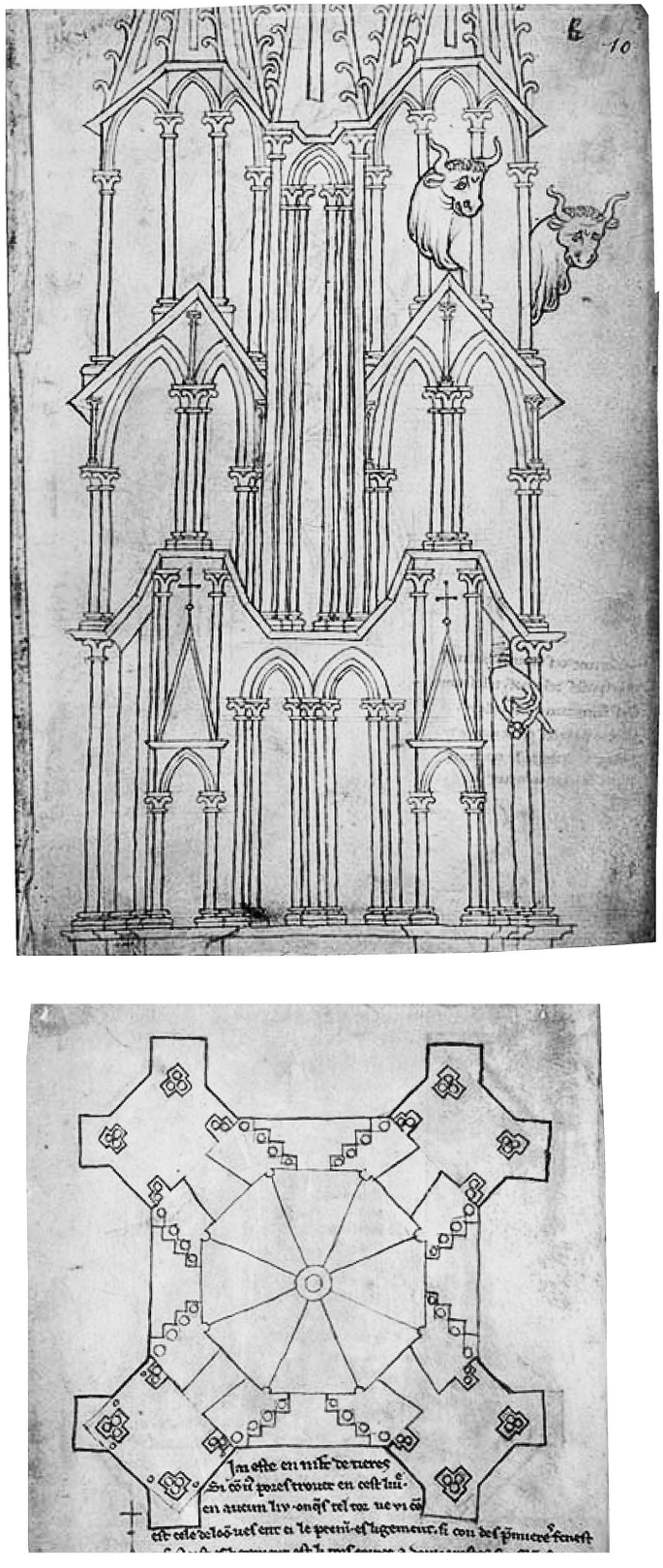

Figure 0.1
Villard de Honnecourt, plan (below) and impressionistic elevation (above) of a tower from Laon Cathedral, from the Portfolio, folios 9v and 10r, respectively. The two drawings have here been set to a common scale, so that their architectural elements align, although the elevation was drawn to a larger scale.

itinerant artist with a lively interest in architecture.[5] His commentaries, which he probably dictated to a scribe rather than writing himself, are interesting and wide ranging, but they certainly do not provide anything like a coherent explanation of Gothic design practice. Since Villard's portfolio was essentially a unique one-off production, one that came to prominence only following its publication in the mid-nineteenth century, it cannot readily be seen as representative of a larger discourse within Gothic workshop culture.

Less charismatic than Villard's portfolio, but more typical of the medieval documentary tradition, are the expense reports and accounts of meetings held by the fabric committees overseeing major Gothic architectural projects. Such unillustrated records, when they survive, can provide a wealth of information useful for analyzing construction procedures, dating building campaigns, and placing them in their social context, but they rarely reveal much about the Gothic design process *per se*. The extensive fabric accounts from the cathedral of Milan, however, deserve special mention in this context, because they have often been invoked to help explain Gothic design practice in general. The situation in Milan, though, was anything but typical. The cathedral, begun in 1386, was unusual both in its grandiose scale and in the fact that its construction brought together designers from both sides of the Alps, who by that time had begun to work in very different architectural traditions.[6] The northerners argued that the building should conform fully to the Gothic cathedral type that had emerged in France, with slender proportions, large windows, deep buttresses, and small capitals. The Italians, by contrast, had never fully accepted the premises of northern Gothic design. The Milanese thus continued to take inspiration from their local Romanesque tradition, even as they sought rhetorical support from sources as diverse as Aristotle and the re-emergent classical idea that a column should have the proportions of the human body. The debates between the northerners and the Milanese were often acrimonious, and the building that resulted from their work must be understood as an unusual hybrid, rather than a normative specimen of a single coherent design tradition.

The Milan fabric accounts have, nevertheless, had a strong influence on discussions of the Gothic approach to geometry, especially since they provide a frequently cited source for the phrases "*ad quadratum*" and "*ad triangulum*." Heinrich Parler, a German consultant active at Milan, recommended that the cathedral should be designed "*ad quadratum*" or "to the square," meaning in this context that the total height of its main vessel should equal the combined width of the vessel and its aisles. "*Ad triangulum*," by analogy, has often been used to describe buildings in which the height of the main vessel equals the height of an equilateral triangle whose base spans the width of the vessel and its aisles. Some Gothic buildings actually do incorporate these proportions, as the discussion of Strasbourg Cathedral in Chapter 2 will demonstrate. At Milan, however, this purely

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[5] This argument has been made most forcefully and consistently by Carl Barnes. For his most thorough treatment, see *The Portfolio of Villard de Honnecourt: A New Critical Edition and Color Facsimile* (Farnham, 2009). Barnes's study effectively supersedes Hans R. Hahnloser's *Villard de Honnecourt: Kritische Gesamtausgabe des Bauhüttenbuches ms. fr. 19093 der Pariser Nationalbibliothek* (Graz, 1972).

[6] This debate is the subject of a series of classic articles by Paul Frankl, "The Secret of the Medieval Masons," *Art Bulletin*, 27 (1945): 46–60; Erwin Panofsky, "An Explanation of Stornaloco's Formula," *Art Bulletin*, 27 (1945): 61–4; and James S. Ackerman, "'Ars Sine Scientia Nihil Est': Gothic Theory of Architecture at the Cathedral of Milan," *Art Bulletin*, 31 (1949): 84–111. More recent discussions of this literature are usefully summarized in Valerio Ascani, *Il Trecento Disegnato*: *Le basi progettuali dell'architettura gotica in Italia* (Rome, 1997), esp. pp. 36–43. See also Chapter 6, below.

geometrical format was quickly abandoned. Already in 1390 the Italian mathematician Gabriel Stornaloco was called in to provide a revised building section, one in which a simple grid of modules would provide a close approximation to the proportions of the equilateral triangle as determined by geometry. By 1400, moreover, this scheme was rejected in favor of one with a lower overall height, in which the proportioning system changed midway up the elevation from Stornaloco's modular system to one governed by horizontally oriented 3-4-5 right triangles. To the extent that its construction involved repeated changes of plan, the case of Milan actually was typical, since such changes were frequent in Gothic workshops. Even in drawings and in unified building campaigns, however, simple schemes based solely on the square or equilateral triangle were rare. By themselves, therefore, the terms "*ad quadratum*" and "*ad triangulum*" provide only an impoverished binary palette of terminology, one that cannot capture the subtlety and detail characteristic of Gothic design in all its shadings.

In the fifteenth century, at last, a few northern Gothic designers began to record their methods in written texts. Increasing literacy rates facilitated this development, but a more important factor was probably the authors' desire to demonstrate the legitimacy of Gothic architectural practice, which was beginning to be challenged by the new fashion for classical design spreading from Renaissance Italy.[7] By the final decades of the century, the invention of the printing press had permitted many educated patrons and builders north of the Alps to gain some familiarity with treatises crucial to classical architectural theory, including most notably *De Architectura*, by the ancient Roman architect Vitruvius, and *De Re Aedificatoria*, by his Renaissance successor Alberti. From a strictly Vitruvian or Albertian perspective, the Gothic tendency to stretch columns heavenward could only be condemned as a deviation from "correct" canonical models based on the proportions of the human body. The creative freedom Gothic designers enjoyed, more generally, could only be deplored as license, while the complexity and virtuosity of Gothic design could be criticized for departing from the clear formal order of classicism.

Gothic designers never managed to mount an effective textual challenge to the emergent Renaissance critique of Gothic architecture, and their few published attempts to explain their methods can most charitably be described as underwhelming. Two small booklets, Matthäus Roriczer's *Büchlein von der Fialen Gerechtigkeit*, of 1486, and Hans Schmuttermayr's roughly contemporary *Fialenbüchlein*, deal only with the design of pinnacles and gablets.[8] Roriczer later published a short booklet called *Geometria Deutsch* demonstrating several basic geometrical constructions, and he may have intended to expand his explicitly architectural discussion, but he certainly left many important design issues unaddressed in his writings.[9] Roriczer's silence on so many topics is particularly frustrating because his position as master of the prestigious Regensburg

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[7] The northern Gothic writings are well discussed in Ulrich Coenen, *Die spätgotischen Werkmeisterbücher in Deutschland* (Munich, 1990). For the possible impact of Italian theory, see Paul Crossley, "The Return to the Forest," *Kunstlerlicher Austausch* (Berlin, 1992): 71–90; and Ethan Matt Kavaler, "Architectural Wit," in Matthew Reeve (ed.), *Reading Gothic Architecture* (Turnhout, 2007), pp. 130–50.

[8] See Coenen, *Die spätgotischen Werkmeisterbücher*; and Lon Shelby, *Gothic Design Techniques* (Carbondale, 1977).

[9] A recent attempt to infer larger lessons from Roriczer's writings is Wolfgang Strohmayer, *Das Lehrwerk des Matthäus Roriczer* (Hürtgenwald, 2004); this attempt is critiqued in Stefan Holzer's review of Strohmayer's book in *Kunstform*, 7 (2006), Nr.03, www.arthistoricum.net/index.php?id=276&ausgabe=2006_03&review_id=10507.

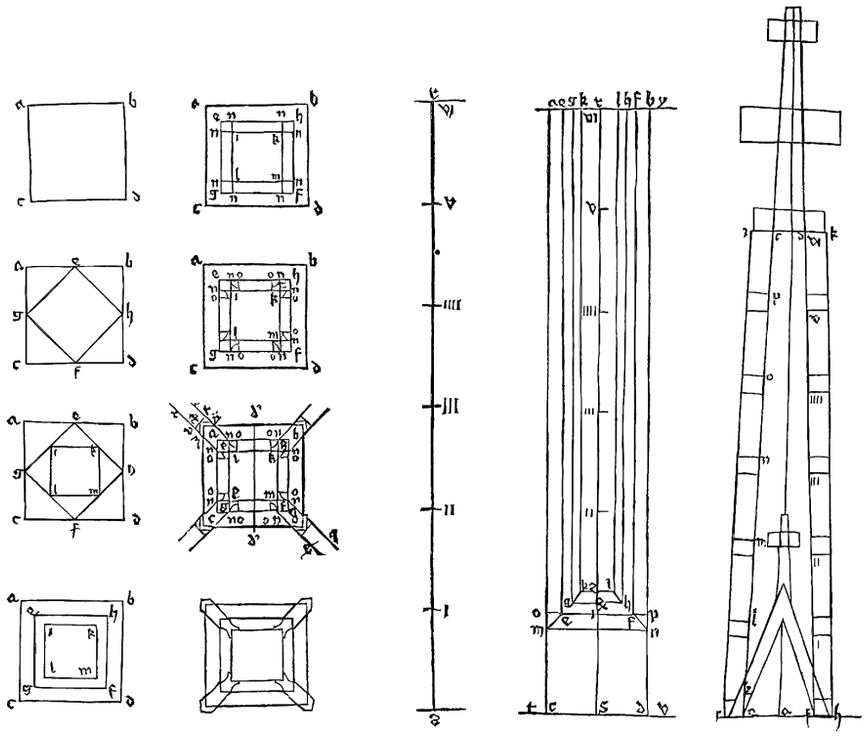

Figure 0.2 Successive stages in pinnacle design, from Roriczer, *Das Büchlein von der Fialen Gerechtigkeit*, 1486, arranged by author

Cathedral workshop qualified him well to discuss all the major issues of Gothic design. It is interesting and significant, however, that both he and Schmuttermayr chose to focus on pinnacles as paradigmatic products of the Gothic design method. Both authors agree that the first step in designing a pinnacle should be to establish a square as its basic groundplan. Next, a series of progressively smaller rotated squares should be inscribed within the original square, in a sequence often called "quadrature" (Figure 0.2). Further permutations of these figures, easily accomplished with the compass and straightedge, sufficed to determine the complete groundplan of the pinnacle. The elevation of the pinnacle was then determined by stacking up a series of modules based on the groundplan. This process of extrusion from the groundplan into the third dimension, which German authors call *Auszug*, or "pulling out," was fundamental to the Gothic design method as a whole. Roriczer himself hints that something more general than pinnacle construction is at stake in his booklet. On its first page, he explains to his learned patron, Wilhelm von Reichenau, the Bishop of Eichstätt, that his writings will "explain something of the art of geometry, beginning with first steps in extruding stonework . . . using proper measures determined with a compass."[10] As the case studies in the subsequent chapters of this book will demonstrate, geometries like those described by Roriczer and Schmuttermayr occur in a wide variety of Gothic drawings, not just those depicting tabernacles, church spires,

10 Shelby, *Gothic Design Techniques*, p. 83.

and other pinnacle-like structures, but also those depicting complete churches and their buttressing systems. These authors may well have chosen to focus on pinnacles for basically pedagogical reasons, thinking that this simple example could clarify design principles of wide applicability, but the seeming narrowness of their topic surely diminished the impact of their writings.

Roriczer, Schmuttermayr, and the few late medieval authors who attempted to provide more comprehensive pictures of Gothic design practice encountered a fundamental problem, one that has bedeviled all similar projects up to the present day—namely, the fact that the geometrical logic of Gothic architecture is hard to explain in words. Because Gothic design conventions govern the rules of the process more than the shape of the final product, the spatial relationships between building components varied far more widely in Gothic than in classical architecture. This, in turn, means that precision can be achieved only with explicit description, rather than with allusions to venerated prototypes. Roriczer, who sought to explain only a simple pinnacle, labeled every single point in his illustrations, describing the successive steps of the design process in numbing detail. Having established the basic square abcd in step one, for example, he explains the next step as follows: "Divide the distance from a to b into two equal parts, and mark an e at the midpoint. Do the same from b to d and mark an h; from d to c and mark an f; from c to a and mark a g. Then draw lines from e to h, h to f, f to g, and g to e, as in the example of the figure drawn hereafter . . ."[11] Despite its tediously explicit detail, Roriczer's text is all but unintelligible without reference to his illustrations.

Three decades after the publication of Roriczer's booklet, the noted Heidelberg court architect Lorenz Lechler tried to explain Gothic design more quickly and economically in his *Unterweisungen*, a more comprehensive compendium of architectural advice for his son Moritz.[12] Although Lechler's known architectural works, such as the sacrament house of S. Dionys in Esslingen, are formidable in their geometrical complexity, his writings present mostly short rules of thumb based on simple arithmetical ratios. He recommends, for example, that side aisle spans should be one half as great as the free span of the main central vessel, which he takes as his fundamental module. The thicknesses of the walls and piers, he suggests, should equal one tenth of this module. The capitals of the main vessel should fall either one module, or alternatively one and a half modules, above the floor. Lechler's short modular recipes are less painful to read than Roriczer's detailed geometrical instructions, but they ultimately prove frustrating, since they fail utterly to explain the origins of the complex dynamic forms that make German late Gothic design so interesting. These examples, moreover, are unillustrated, at least in the three surviving manuscripts of the *Unterweisungen*. These manuscripts do, however, include several illustrations showing how combinations of geometrical and arithmetical subdivision could be used to generate the cross-sections of window mullions, taking the wall thickness in this case as the given module (Figures 0.3 and 0.4). Here, one begins to catch a glimpse of how Gothic designers derived the details of their buildings from the dimensions of the larger structure, but it is hard to extrapolate from these examples to get a satisfying picture of the overall design process.

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11 Quoted from Shelby, *Gothic Design Techniques*, p. 85.

12 Coenen, *Die spätgotischen Werkmeisterbücher*, pp. 15–25 and 146–52.

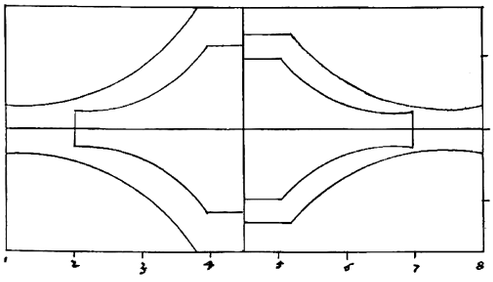

Figure 0.3 Cross-sections of large and small mullions, from Lechler's *Unterweisungen*.

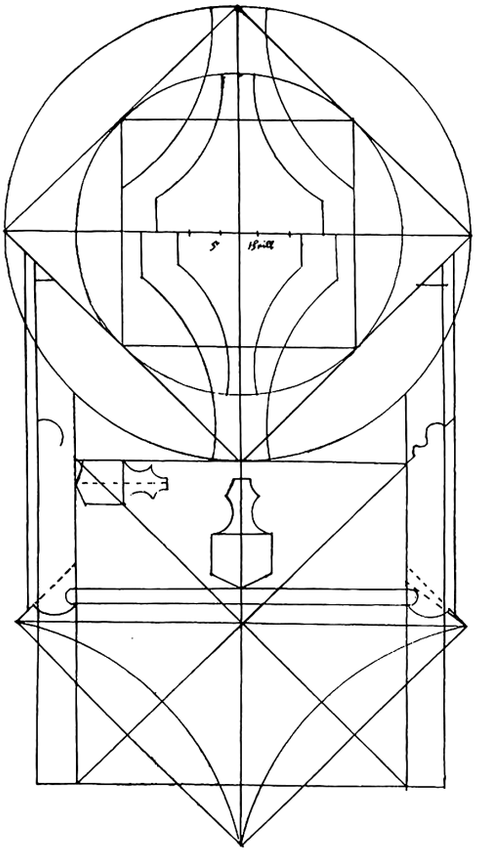

Figure 0.4 Cross-sections of mullions and moldings in relation to wall thickness, from Lechler's *Unterweisungen*.

Before going on to consider the reception of the late Gothic design handbooks, it is worth pausing briefly to ponder the ways in which geometrical and arithmetical thinking could intersect in the Middle Ages, since this topic will arise repeatedly in subsequent chapters. Medieval masons and church designers, of course, approached geometry with a view to its practical application in architecture. Unlike modern mathematicians, and

unlike the ancient Greeks, they cared little for analytical logic and the construction of rigorous proofs. The distinction between rational and irrational numbers, which mattered greatly to Greek theorists, was not terribly important to Gothic builders, who frequently used arithmetical ratios to approximate the irrational quantities that result from geometrical operations.[13] This can be seen not only in Stornaloco's intervention at Milan, but also in the illustrations from Lechler's *Unterweisungen*, here reproduced as Figures 0.3 and 0.4.[14] The two nested forms in Figure 0.3 represent the cross-sections of the large and small window mullions in a given church choir. As the numbered hash marks along the side of the figure show, the smaller should be five sevenths as tall as the larger. The recurrence of these profiles in the upper half of Figure 0.4, however, makes clear that this 5:7 ratio is actually just an approximation to the 1:√2 ratio arising from square rotation and quadrature. This example thus differs in an important sense from Lechler's previously cited recommendation that wall thicknesses equal a tenth of the main vessel width, a rule in which the design method was truly and exclusively modular. Lechler and his Gothic colleagues, however, never systematically distinguished between the geometrical and arithmetical design modes. Indeed, their few surviving writings are strikingly unsystematic altogether. Like many other medieval technical texts, in fact, they are essentially just compilations of recipes, rather than polished treatises with clear organization and argument structure.

Since the Gothic design process was inherently difficult to describe, and since the few late medieval authors who tried to do so lacked rhetorical sophistication, it is perhaps unsurprising that their work was largely swept aside by the wave of theoretical writings on architecture coming from Renaissance Italy. The publication of Serlio's lavishly illustrated seven-volume *Libri d'Architettura*, in particular, helped to establish a taste for classical design even in northern courts where Gothic architecture had formerly predominated. By the middle of the sixteenth century, in fact, classical art and architecture had largely taken over in Europe's leading centers of fashion. Many different factors seem to have contributed to the rapid collapse of the Gothic tradition, as the final chapter of this book will explain, but the great rhetorical appeal of Renaissance writings was surely one of the most important. It is possible to argue, in fact, that Renaissance architecture, like so many new commodities, achieved popularity more because of effective marketing than because of the product's inherent quality. The classical critique of Gothic art, which had only begun to emerge in the fifteenth century, achieved canonical status in Vasari's *Lives*, one of the earliest and most influential histories of art ever written. Vasari's condemnation of Gothic

[13] For the general principles of this conversion process, see Peter Kidson, "A Metrological Investigation," *Journal of the Warburg and Courtauld Institutes*, 53 (1990): 71–97. For an excellent example of how this transition between geometrical schemes and arithmetical approximations could be applied in medieval building, see idem, "The historical circumstance and the principles of the design," in Thomas Cocke and Peter Kidson (eds), *Salisbury Cathedral: Perspectives on the Architectural History* (London 1993), pp. 31–97, esp. 62–75. Kidson explicitly points out (p. 62) that the use of rational approximations was widespread prior to the advent of architectural drawing. Once drawing became commonplace, however, it surely would have been simpler for draftsmen to construct quantities like √2 and √3 by using their compasses to make squares and triangles, rather than by using arithmetical approximations. So, while the approximations discussed by Kidson were likely used on building sites throughout the Gothic era, emphasis in the present study will remain on the geometrical relationships that the draftsmen developed on their parchments.

[14] These illustrations are carefully discussed in Werner Müller, *Grundlagen gotischer Bautechnik: ars sine scientia nihil* (Munich, 1990), pp. 90–94.

architecture as a barbaric and lawless deviation from classical correctness continues to color popular perceptions of art history still today, even though his association of this architecture with the Goths who sacked Rome has long been recognized as a polemical myth.[15] The triumph of classical art in the Renaissance, of course, was not absolute, and the line between Gothic and Renaissance visual culture could be surprisingly blurred at times. In 1521, for example, the Milanese Cesare Cesariano published a pioneering Italian-language edition of Vitruvius's *De Architectura* featuring illustrations not only of classical buildings and their components, but also of Milan Cathedral with an idealized "*ad triangulum*" overlay. In Germany and England, moreover, remnants of Gothic architectural culture survived as a craft tradition even into the seventeenth and eighteenth centuries.[16] By the time interest in Gothic architecture revived in the years around 1800, however, a great deal about genuine medieval design practice had been forgotten.

Over the past two centuries, many attempts have been made to rediscover the lost "secrets" of the Gothic design method. Progress in this direction, however, has not been steady. Some of the earliest writings of the Gothic revival already contain valuable insights, while some recent publications, unfortunately, still present fantastic ideas unsupported by strong evidence. The young Goethe, in his path-breaking 1773 essay "*Von Deutscher Baukunst*," helped to set the stage for later authors by praising the west façade of Strasbourg Cathedral as a highly ordered work of genius. He recognized, in other words, that classical dismissals of Gothic architecture as chaotic were unfair. Subsequent attempts to characterize Gothic architectural order have run the gamut from impressionistic to analytical. The Romantic writers Georg Forster and Friedrich Schlegel compared Gothic buildings, respectively, to ancient forests and to mineral crystals. These admittedly vague comparisons actually have some merit, because, as the following chapters will demonstrate, Gothic design really does involve sequences of geometrical operations that have more in common with organic and crystalline growth than they do with the more narrowly circumscribed modularity of most post-medieval architecture. In the early nineteenth century, meanwhile, scholars and antiquarians began to republish authentic Gothic documents, including the portfolio of Villard de Honnecourt, in France, and the later design booklets written by Roriczer, Lechler, and their colleagues, in Germany. Such publications, together with the careful study of original Gothic buildings in the course of restoration, helped to make the nineteenth-century Neo-Gothic era a highly productive time for research into Gothic design practice. By the middle of the century, therefore, a sizable literature on Gothic geometry and proportioning had begun to emerge, but the results of this research remain controversial.[17]

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[15] Particularly influential contributions to this literature are usefully summarized in Paul Frankl, *The Gothic: Sources and Literary Interpretations through Eight Centuries* (Princeton, 1960). See also Annemarie Sankovitch, "The Myth of the Myth of the Medieval," *RES: Anthropology and Aesthetics,* 40 (2001): 29–50.

[16] For a good discussion of sixteenth- and seventeenth-century German technical drawings of residually Gothic style, see Müller, *Grundlagen gotischer Bautechnik*, pp. 31–44. Müller also cites the *Vitruvius Deutsch*, by Rivius, a near-copy of Cesariano's Italian Vitruvius, on pp. 41–53.

[17] See, for example, Friedrich Hoffstadt, *Gotisches ABC-Buch, das ist: Grundregeln des gothischen Styls für Künstler und Werkleute* (Frankfurt, 1840), or Carl Alexander Heideloff, *Die Bauhütte des Mittelalters* (Nuremberg, 1844).

Methodological Problems in the Study of Gothic Geometry

Three closely linked methodological problems have undercut the authority of most publications on Gothic geometry: imprecision, ambiguity, and wishful thinking. Geometrical imprecision of one important sort arises when the object studied by the modern scholar differs in its proportions from the original designer's intentions. Most studies of Gothic geometry have been based on modern survey drawings of the buildings in question, which can be imprecise in two ways: the modern drawing may not faithfully reflect the shape of the building; and the building, in turn, may not faithfully reflect the designer's vision if errors or changes were introduced in the construction process. Another sort of imprecision can arise in the testing of geometrical hypotheses if, for example, the testing method involves drawing candidate lines manually across the underlying survey drawing. Even very careful draftsmen using rulers and protractors can drift slightly off the intended angle, and this can create large dimensional differences if the line grows long. When the underdrawings are small and the tip of the drafting instrument is large, even line thickness can become a source of imprecision.

Ambiguity in proportional studies often stems from uncertainty about which points in the building were geometrically meaningful to the original designer. Should the width of an aisle, for example, be measured to the inner wall surface, the outer wall surface, or the centerline in between? Should the height of a nave be measured to the top of the vault keystones, or to the top of the gutteral walls? Such questions can be multiplied almost *ad infinitum*, unless some external constraint is brought to bear. With so many degrees of freedom in the geometrical system, it becomes possible to generate armies of trial lines, adjusting their position and orientation until they appear to hit something interesting.

Because imprecision and ambiguity have been so prevalent in studies of Gothic proportions, especially those undertaken before 1950 or so, wishful thinking has played an important role in shaping claims about the geometrical character of Gothic design. Many scholars, having found apparent evidence for one kind of proportioning system in a given Gothic building, sought to demonstrate its more widespread application by adding trial lines to drawings of other buildings. The turn-of-the-century architectural historian Georg Dehio, for example, published a series of graphics purporting to show that medieval church sections of all sorts depended on "*ad triangulum*" schemes like those briefly entertained at Milan.[18] Other authors argued for the widespread application of different figures, such as the 45-degree isosceles triangle, the pentagon, or its famous cousin, the so-called Golden Section, which is the ratio Ø satisfying the harmonic equation Ø=1/(Ø-1).[19] These figures were sought out in Gothic architecture not so much

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[18] Georg Dehio, *Untersuchungen über das gleichseitige Dreieck als Norm gotischer Bauproportionen* (Stuttgart, 1894); idem, "Zur Frage der Triangulation in der mittelalterlichen Kunst," *Repertorium für Kunstwissenschaft*, 18 (1895); and Dehio and G. von Bezold, *Die kirchliche Baukunst des Abendlandes* (Stuttgart, 1901). E.-E. Viollet-le-Duc also thought triangular proportions contributed to stability. See his article "Proportion," in *Dictionnaire Raisonné de l'architecture francaise* (Paris, 1858–68), vol. 7, esp. pp. 532–42.

[19] The 45-degree triangle was championed by Karl Alhard von Drach in *Das Hüttengeheimnis von gerechten Steinmetzgrund* (Marburg, 1897). Pentagons figure prominently in Karl Witzel, *Untersuchungen über gotische Proportiongesetze* (Berlin, 1914). Julius Haase, meanwhile, claimed that the Golden Section established the location of the crossing spire of Cologne Cathedral, in "Der Dom zu Köln am Rhein in seinen Haupt-Maßverhältnisse auf Grund der Siebenzahl und der Proportion des goldenen Schnitts," in

because the medieval evidence suggested their presence, but because they were associated, in the researchers' minds, at least, with prestigious theories of beauty and meaning.[20] The Golden Section has a distinguished ancient pedigree, having been known to both the Egyptians and the Greeks. Regular polygons, meanwhile, evoke the surfaces of the Platonic solids, and the number of sides on each figure can readily be associated with numerological theories, both ancient and medieval. Written sources from the Gothic era demonstrate that clergymen often did use numerological references to link architecture with meaning, as when Abbot Suger of Saint-Denis compared the twelve inner columns of his church with the twelve apostles, but such *post-facto* associations provide indirect evidence, at best, for the working methods of the church designers themselves.

In seeking to connect Gothic architecture with prominent aesthetic and symbolic systems, therefore, many twentieth-century researchers drew dense networks of lines based on their pet systems all over drawings of Gothic buildings; Figure 0.5 shows a typical example, taken from Fredrik Macody Lund's 1921 *Ad Quadratum*, in which a system based on the pentagram is imposed on the cross-section of Notre-Dame in Paris.[21] Most of the key points in the diagram lie below ground or in the air, rather than on crucial elements of the building, which is represented in accord with a conjectural reconstruction by the nineteenth-century architect Viollet-le-Duc that has been rejected by subsequent scholarship. Some work in this genre may include valuable observations, but the imprecision and ambiguity inherent in the testing process make it all but impossible to distinguish the justified conclusions from the flights of interpretive fancy. Much of the early work in this field, therefore, was dismissed in a rigorous critical study published by Walter Thomae in 1933.[22] Three decades later, in his magisterial review of writings on the Gothic period, Paul Frankl wrote in apparent frustration that "The question of what is actually gained by such research becomes urgent. There can be no doubt that Gothic architects made use of triangulation and the like, but the excogitated networks made up of hundreds of lines to determine all points has not been proved and is probably undemonstrable and unlikely."[23] Perhaps the most devastating critique of this geometrical research tradition came in the early 1970s from Konrad Hecht, whose writings will be considered in detail below. Despite such skepticism in the academy, a wide range of authors continue to publish more or less plausible theoretically driven geometrical overlays even

Zeitschrift für Geschichte der Architektur, 5 (1911/1912); and Otto Kletzl claimed that the Golden Section established the relative heights of the tower and spire in Freiburg, in "Zwei Plan-Bearbeitungen des Freiburger Münsterturms," *Oberrheinische Kunst*, 7 (1936): 15. This claim was repeated by Adolf Wangart in *Das Münster in Freiburg im Breisgau im rechten Maß* (Freiburg, 1972).

[20] This is not to say, of course, that such figures were never used in Gothic design. Stephen Murray and James Addiss have effectively demonstrated the use of the Golden Section in "Plan and Space at Amiens Cathedral: With a New Plan Drawn by James Addiss," *Journal of the Society of Architectural Historians*, 49 (1990): 44–66. Peter Kidson, meanwhile, has documented the use of the Golden Section at Salisbury, and he has gone on to show that good rational approximations for √2, √3, √5, and Ø were known throughout the Middle Ages. See "The Historical Circumstance and the Principles of the Design." These rigorous modern studies are supported by a degree of archaeological and documentary evidence unseen in the more speculative publications of the early twentieth century.

[21] Fredrik Macody Lund, *Ad Quadratum* (London, 1921).

[22] Walter Thomae, *Das Proportionswesen in der Geschichte der gotischen Baukunst und die Frage der Triangulation* (Heidelberg, 1933).

[23] Paul Frankl, *The Gothic*, p. 721.

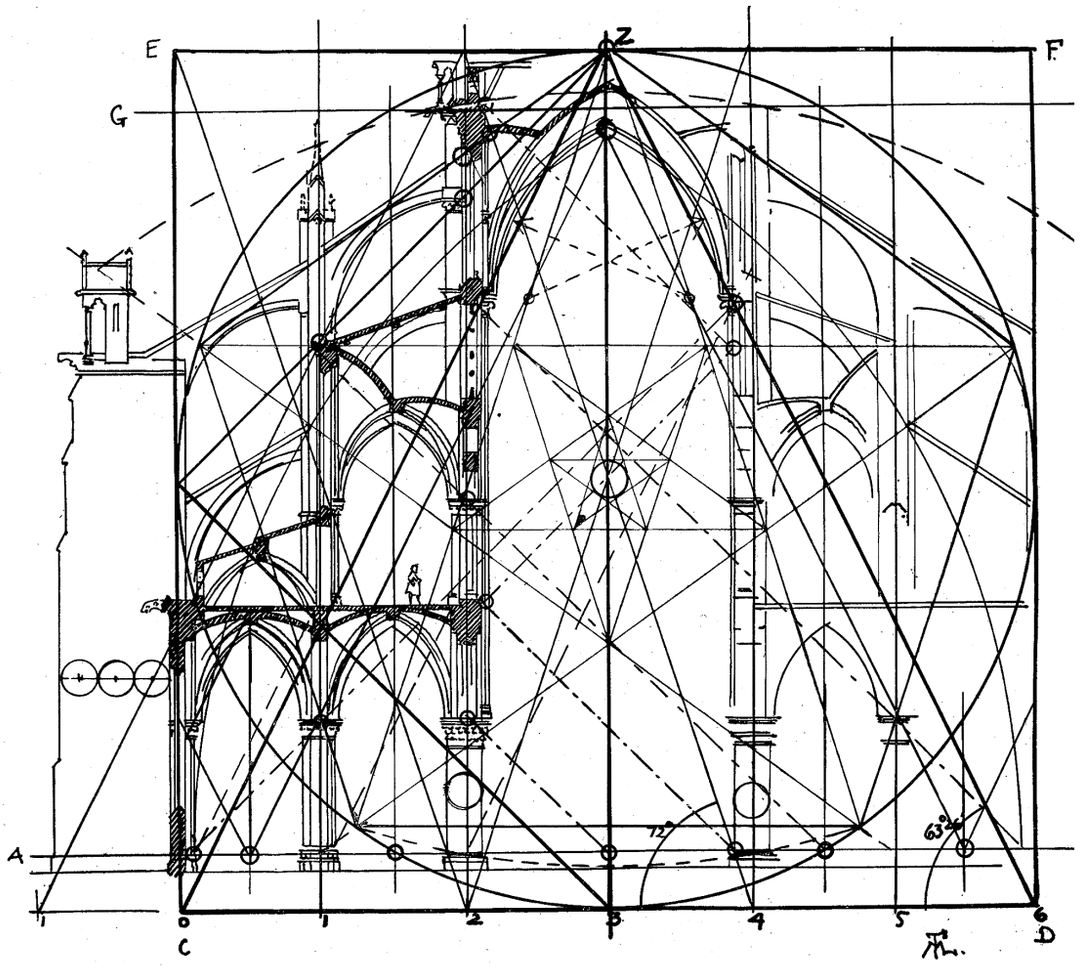

Figure 0.5 Hypothetical geometrical overlay drawn by Fredrik Macody Lund on an equally hypothetical cross-section of Notre-Dame in Paris, which was originally proposed by Viollet-le-Duc. Source: Lund, *Ad Quadratum*, Figure 52, p. 36.

today.[24] By themselves, however, such publications are unlikely to dispel the lingering distrust that many scholars feel for geometrical studies of Gothic design.

The study of original Gothic architectural drawings offers at least a partial way out of this impasse, because this approach avoids many of the imprecision and ambiguity problems outlined above. Drawings, unlike buildings, can be measured easily and directly, and their proportions are uncompromised by errors introduced in the construction process. The presence of compass prick points and construction lines in the drawings, moreover, helps to solve the ambiguity problem by suggesting which points and lines

[24] On the more methodologically rigorous side, Nigel Hiscock's *The Wise Master Builder* (Aldershot, 2000), deserves mention. Not everyone will be convinced by Hiscock's argument for the relevance of Platonic geometry for medieval architecture, but he does carefully describe his research method and the precision of his claimed results. More troubling is Thierry de Champris, *Cathédrales, le verbe géométrique* (Paris and Brussels, 1994), which purports to connect the geometry of French cathedral façades with the "paths of wisdom" leading through the Old and New Testaments to Pythagorean Greece and ancient Egypt.

the original designers actually used in developing their schemes. These factors make Gothic drawings ideal subjects for geometrical study. To date, however, surprisingly little progress has been undertaken in this direction. Most studies of geometry have involved buildings rather than drawings, while most studies of the drawings, conversely, have sidestepped geometrical questions. Three scholars who did begin to apply geometrical methods to the study of Gothic drawings were Otto Kletzl, who wrote a series of articles on the subject in the 1930s and 1940s[25]; Walter Ueberwasser, who considered the work of Villard de Honnecourt in the course of his roughly contemporary studies of artistic proportion[26]; and Maria Velte, whose short 1951 book *Die Anwendung der Quadratur und Triangulatur bei der grund- und aufrißgestaltung der gotischen Kirchen* remains one of the most widely cited studies in the field.[27] Velte's application of triangulation schemes to building sections owed much to Dehio, but her application of quadrature schemes to original plan drawings of major Gothic towers promised a greater degree of analytical rigor than most of what had come before. In spirit and method, in fact, these analyses are among the closest precursors to those presented in the following chapters of this book. Velte tended, however, to adopt an overly rigid view of geometrical systems like quadrature, failing to grasp the quasi-organic flexibility of the Gothic design system. Her proposed geometrical systems, therefore, sometimes align only very imperfectly with the details of the original plans, a shortcoming that was not lost on skeptics such as Hecht. In the years around 1970, François Bucher began to outline an ambitious research program that would tie together geometrical analysis with careful consideration of Gothic drawings. He published several valuable articles that evocatively describe the basic geometrical principles of Gothic design, but he presented little in the way of new data, and the promise of his program remains largely unrealized.[28] More progress along the lines Bucher foresaw probably would have been made in the past four decades, if Konrad Hecht's work had not dramatically altered the scholarly climate.

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[25] Otto Kletzl, "Werkrißtypen deutscher Bauhüttenkunst," *Kunstgeschichtliche Gesellschaft Berlin, Sitzungberichte*, (1937–38): 20; "Ein Werkriss des Frauenhauses in Strassburg," *Marburger Jahrbuch für Kunstwissenschaft*, 11–12 (1938–39): 103–58; *Plan-Fragmente aus der deutschen Dombauhütte von Prag in Stuttgart und Ulm* (Stuttgart, 1939); "Die Kressberger Fragmente: Zwei Werkrisse deutscher Hüttengotik," *Marburger Jahrbuch für Kunstwissenschaft*, 13 (1944): 129–70. Also relevant are his analyses of the Freiburg tower in "Zwei Plan-Bearbeitungen des Freiburger Münsterturms," *Oberrheinische Kunst*, 7 (1936): 14–35; and his critical review of Thomae's *Das Proportionenwesen in der Geschichte der gotischen Baukunst*, in *Zeitschrift für Kunstgeschichte*, 4 (1935): 56–63.

[26] Walter Ueberwasser, "Nach rechtem Maß: Aussagen über den Begriff des Maßes in der Kunst des XIII.–XVI. Jahrhunderts," in *Jahrbuch des preußischen Kunstsammlungen* 6 (1935): 250–61 and Abb. 7 esp.; idem, "Der Freiburger Münsterturm im 'rechten Maß'," *Oberrheinische Kunst* 8 (1939): 25–32.

[27] Maria Velte, *Die Anwendung der Quadratur und Triangulatur bei der grund- und aufrißgestaltung der gotischen Kirchen* (Basel, 1951). Velte's work was very positively reviewed by James Ackerman in *Art Bulletin*, 35 (1953): 155–7.

[28] François Bucher, "Design in Gothic Architecture: A Preliminary Assessment," *Journal of the Society of Architectural Historians*, 27, (1968): 49–71; idem, "Medieval Architectural Design Methods, 800–1560," *Gesta*, 11 (1972): 37–51; idem, "Micro-Architecture as the 'Idea' of Gothic Theory and Style," *Gesta*, 15/1–2 (1976): 71–91; idem, *Architector, The Lodge Books and Sketch Books of Medieval Architects* (New York, 1979). This last, which considered only the work of Villard de Honnecourt, Hans Böblinger, and the so-called Master WG, was intended to be the first of a multi-volume set discussing many of the major drawings of the Gothic period, but the anticipated sequels were never published.

Konrad Hecht's problematic critique of geometrical research

Hecht's work occupies a singular place in the historiography of Gothic design, because his critique of geometrical research was so aggressive, comprehensive, and densely argued that it nearly stopped the field in its tracks, with particularly demoralizing impact on the drawing-related investigations that Kletzl, Velte, and Bucher had begun to explore. Hecht began, it seems, with the best of intentions, seeking to restore historical realism to a field in which fantastic hypotheses had grown unchecked.[29] He opened his major publication in the field, *Maß und Zahl in der gotischen Baukunst*, with a rigorous critical review of previous literature, amply demonstrating the arbitrariness, implausibility, and mutual incompatibility of much that had come before.[30] Next, he set out to demolish all the successive attempts to illustrate geometrical proportioning systems in the tower of Freiburg Minster, which had become something of a hobbyhorse for writers in the field. Through careful numerical analysis, Hecht showed that imprecision, ambiguity, and wishful thinking do indeed flaw most of these studies.

Reacting against the excesses of purely geometrical design theory, Hecht proposed an even more implausible alternative, suggesting that Gothic designers worked almost exclusively in arithmetical modular fashion. To make this argument, Hecht had to look south of the Alps for evidence, while simultaneously working to diminish the evidence for geometrical design practices in the northern Gothic world. He used documents related to the construction of Milan Cathedral and San Petronio in Bologna to show that module-based thinking was, indeed, crucial for Italian architectural practice in the later Middle Ages. This, however, proves very little about the northern Gothic situation, since Italy had never fully accepted the premises of northern Gothic design. Hecht then attempted to show that module use could provide a better explanation than geometry for certain famous specimens of northern Gothic design, including the portfolio of Villard de Honnecourt, the tower of Freiburg Minster, and the drawings related to the planning of Ulm Minster.[31] In discussing Villard's plan for the tower of Laon Cathedral, for example,

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[29] Other more complex ideological forces may well have informed Hecht's distrust of geometrical explanations for Gothic design. Since the geometrical sophistication of German Gothic design was a source of nationalist pride for authors such as Kletzl who enjoyed favored positions in the Third Reich, this intellectual legacy likely appeared tainted after the Second World War. This would have been particularly the case for Hecht, since he worked at the University of Braunschweig, where a strict and reductive modernism dominated the architecture school in the decades after the war, providing a strong critique of the Reich and its bombastic historicism. On Kletzl's career in the war years, see Adam Labuda, "Das Kunstgeschichtliche Institut an der Reichsuniversität Posen und die „nationalsozialistische Aufbauarbeit" im Gau Wartheland 1939–1945," in Jutta Held and Martin Papenbrock (eds), *Kunstgeschichte an den Universitäten im Nationalsozialismus* (Göttingen 2003), pp. 143–60. On the architecture school in Braunschweig, see *Die Architekturlehrer der TU Braunschweig*, eds Roland Böttcher, Kristiana Hartmann and Monika Lemke-Kokkelink (Braunschweig, 1995).

[30] Hecht's *Maß und Zahl in der gotischen Baukunst* first appeared as three successive issues of *Abhandlungen der Braunschweigischen Wissenschaftlichen Gesellschaft*: 21 (1969), 22 (1970), and 23 (1970). The complete study has been republished as a single volume by Georg Olms Verlag (Hildesheim, 1979). The page numbers cited in the present study are from the more widely available 1979 version.

[31] Hecht's *Maß und Zahl* includes the following passages cited here: the general critique of earlier literature, mostly on pp. 2–60; the critique of geometrical literature on the Freiburg tower in particular, pp. 60–92; the appeal to Italian sources, pp. 130–71; Villard de Honnecourt, pp. 201–17; a modular approach to the Freiburg tower, pp. 334–61; Gothic drawings in general, 381–7; the Ulm elevation drawings in particular, pp. 387–468.

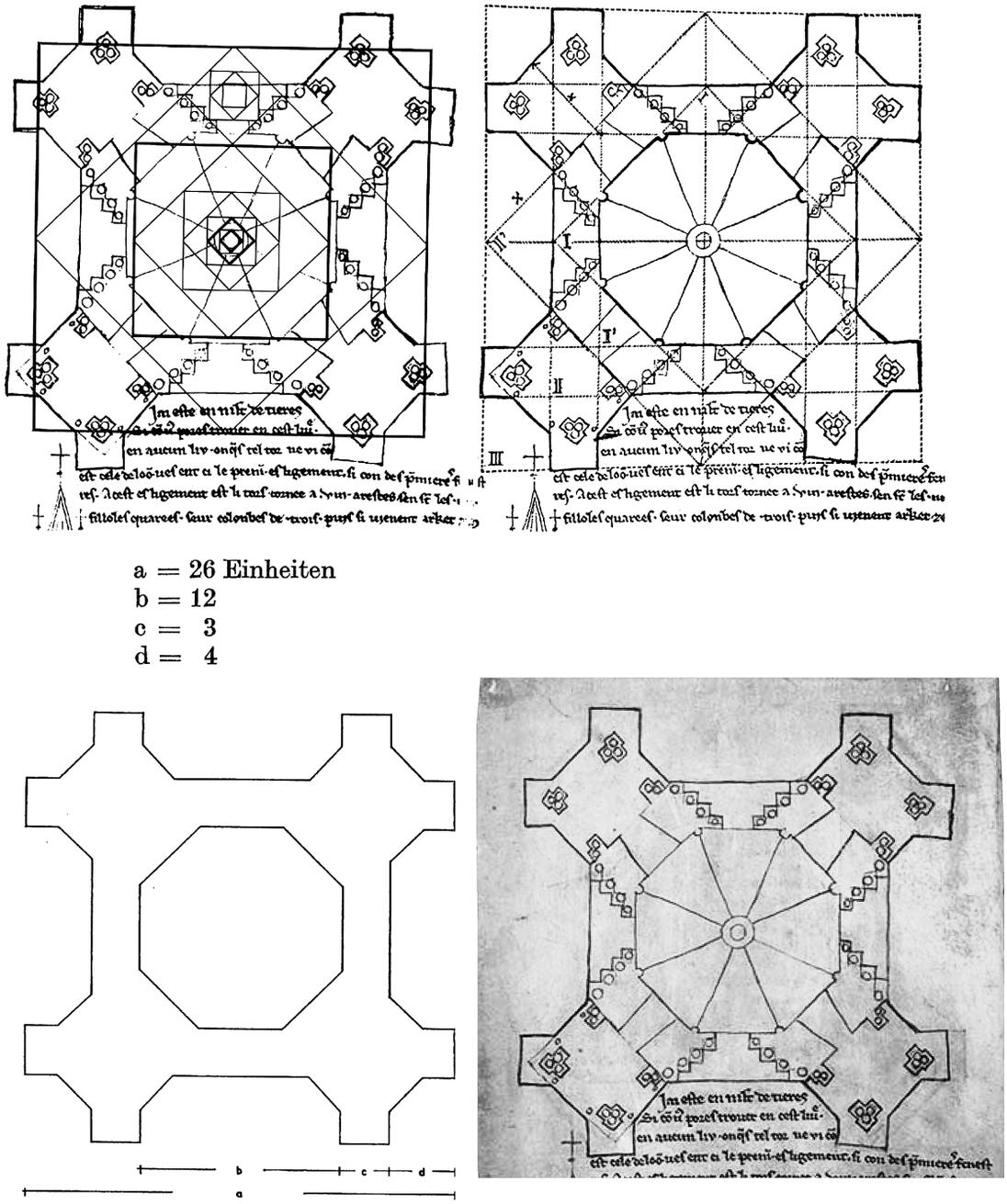

Figure 0.6 Comparison between four different approaches to Villard de Honnecourt's Laon towerplan. The original drawing (as in Fig. 0-1) appears at bottom right; Ueberwasser's geometrical scheme at upper right; Velte's alternative geometrical scheme at upper left, and Hecht's modular description at lower left.

Hecht noted with apparent satisfaction that Ueberwasser and Velte suggested very different geometrical schemae to explain Villard's drawing. He then proposed a purely numerical proportional system which, he claimed, explains the drawing better than either of the geometrical systems (Figure 0.6).[32] In doing so, he failed to admit two key facts: first, that Ueberwasser's scheme actually explains the basic proportions of Villard's drawing quite well; and, second, that his own modular scheme explains very little, since he gave no reason why the tower proportions should involve the modules he proposed. He was, in essence, just presenting a numerical approximation to a set of proportions that could easily have been determined by geometrical means. This, indeed, is the fundamental problem with his overall argument.

Seeking to provide a more convincing demonstration of his theories than analysis of Villard's imprecise sketches can provide, Hecht proposed a modular explanation for the proportions of the Freiburg tower and spire. His discussion appears rigorous at first sight, since it invokes hundreds of careful measurements of the structure, and since it includes meticulous centimeter-by-centimeter error analyses that compare these data with his hypotheses. Upon closer investigation, however, this appearance of rigor dissolves, as it becomes clear that Hecht shrank from obvious conclusions when they challenged his *a priori* assumptions. For example, he refused to admit that the inner layer of the Freiburg tower wall was likely meant to be half the total width of the wall, even though he clearly showed that their two thicknesses were, respectively, 98 and 196 centimeters. Instead of simply observing that $196 = 2 \times 98$, Hecht worked laboriously backwards from his assumption that the tower was laid out in feet and inches, where he claimed that the foot unit in the Freiburg workshop was 31.095 centimeters. On this dubious basis, he concluded, first, that the 196-centimeter wall thickness was actually meant to be 194 centimeters, which would be 6 feet 3 inches according to his system, and second, that the 98-centimeter layer thickness was meant to be 101 centimeters, which would be 3 feet 3 inches in his system.[33] He basically conjured constructional errors out of thin air, in other words, self-consciously shrinking and expanding the building's dimensions to fit them into the Procrustean bed of his modularity thesis. Even after torturing the evidence to fit his preconceptions, though, Hecht could not explain why the Freiburg designer would have chosen these particular lengths. Indeed, he does not even try. And, Hecht effectively limited himself to describing an imagined rectilinear space box around each tower component, rather than considering what the shape and articulation of the component may suggest about its formal logic. Small wonder, since the shapes of the elements in the Freiburg tower clearly reveal the importance of a geometrically based design process, as the analysis in Chapter 2 of this book will show. There is no way to describe these forms, or even the simple octagonal form of the tower core itself, in purely modular terms, a fact that Hecht admitted only very grudgingly. He implicitly acknowledged the importance of geometrical constructions, for example, when he used the proportions of a perfect octagon to compute the ideal side lengths for the Freiburg tower octagon, even though the diagonals in the octagon introduce irrational numbers that can have no place in a purely modular design system. Despite such occasional slips from his own arithmetical orthodoxy, however, Hecht generally remained critical of geometrical explanations for

[32] Hecht, *Maß und Zahl*, pp. 207–10.

[33] Hecht, *Maß und Zahl*, pp. 353, 358, Abb. 81.

Gothic proportion, concentrating his creative energies on the elaboration of complex modular systems.

The final section of Hecht's study, and the one of greatest relevance for this book, discusses Gothic architectural drawings. Here, as usual, he made many important and valid points, while simultaneously managing to ignore the copious evidence for the importance of geometrical design methods in the Gothic era. Hecht correctly observed, for example, that Gothic drawings typically include only a few geometrical construction lines—certainly far fewer than one would expect from the modern authors who support their arguments with dense networks of lines. He went astray, however, in arguing that geometrical proportioning figures were not used by Gothic designers to establish the main outlines of their compositions.[34] As the following chapters will demonstrate, such figures certainly were used, especially in the development of groundplans, but also in the construction of elevations. Compass holes and construction lines bear witness to this in many cases. Their relative paucity in any given drawing seems to reflect both the draftsmen's economy of means, and their desire to keep especially their presentation drawings free of distracting lines that would have no place in the finished building.[35] In some cases, moreover, dimensions may well have been worked out geometrically on a groundplan, before being transferred to the corresponding elevation, obviating the need for many construction lines in the latter.

Hecht took care to consider not just the appearance of Gothic drawings, but also their size. He strongly objected to the popular idea that Gothic designers developed their drawings geometrically starting from baselines of essentially arbitrary scale. He argued, on the contrary, that Gothic drawings originally had simple scale relationships to the buildings they depicted. He claimed, in particular, that these relationships were often based on decimal or duodecimal subdivision. The drawings, in other words, might be smaller than the full-scale buildings by factors such as 10, 20, 30, or 40; or 12, 24, 36, or 48. Such scalings were actually mentioned in medieval documents, including Lechler's *Unterweisungen*, and many surviving drawings at least roughly accord with these proportions, so Hecht's basic claim is probably correct.[36] It seems certain, at any rate, that Gothic builders established the scales of their drawings carefully and systematically.

There are several respects, however, in which Hecht's larger argument about scaling breaks down. First, it is by no means clear that all Gothic drawings were scaled in the simple way he describes. Many of the drawings discussed in this book, and even many of the examples he cites himself, now have scales that differ measurably from those he sees as normative. To explain these discrepancies Hecht invoked the dimensional instability of parchment, which shrinks and expands in response to changes in humidity. Using a complex and rather unconvincing argument based on the statistical analysis of trial dimensions, Hecht concluded that the total shrinkage factors over the centuries might be

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[34] Hecht, *Maß und Zahl*, pp. 370–72.

[35] In the literature on Gothic drawings, a distinction has often been observed between lavishly detailed presentation drawings designed to appeal to patrons, and humbler technical drawings intended only for the use of workshop insiders. This distinction should not be seen as absolute, however, since even the large and impressive Gothic drawings discussed in this book were clearly created using geometrical means precisely like those seen in the smaller workshop drawings.

[36] Hecht, *Maß und Zahl*, pp. 381–2; idem, "Zur Maßstablichkeit der mittelalterlichen Bauzeichnung," *Bonner Jahrbuch*, 166 (1966): 253.

as great as 7 percent. With tolerances this large, it is fairly easy to connect the drawings to one hypothetical scale or another. So, the evidence for consistent decimal and duodecimal scalings is not as precise and powerful as it might be.

Even if it could be demonstrated that the scale of every single Gothic drawing related to that of the real building by a simple integer multiple, just as Hecht proposes, this would not imply that the design method in question was based exclusively on modular rather than geometrical thinking. There is no reason, in principle, why a Gothic draftsman could not work geometrically within the framework of a well-chosen module, or even within a modular array. Such combinations of geometrical and modular proportioning schemes, in fact, appear to have been typical of Gothic design practice, as the following chapters will demonstrate. In this book, with its emphasis on geometry rather than modularity, discussions of scale will figure less prominently than in Hecht's study. This approach is warranted because only differential changes of scale affect geometrical relationships. In most instances discussed in this book, such differential changes are negligible, as one can tell from the fact that compass-drawn circles remain truly circular, for example. Even overall changes of scale, which leave geometry unaffected, appear in directly testable cases to be far smaller than those claimed by Hecht.[37]

Hecht's study culminates with proportional analyses of the elevation drawings from Ulm Minster, analyses that must be criticized as unsatisfying despite their seeming rigor.[38] Hecht approached the drawings in a purely quantitative fashion, transforming these masterpieces of draftsmanship into dense and almost unreadable tables of numbers. This approach can be helpful for the testing of hypotheses, but it has major drawbacks, since it obscures relationships that would be obvious from a more visual perspective. Hecht's decision to consider only the elevation drawings is also problematic, since the proportions of the elevations reflect those that had already been established in the corresponding groundplan drawings, where compass pricks and construction lines clearly attest to the use of geometrical rather than strictly arithmetical planning methods. As Chapter 4 of this book will reveal, in fact, geometrical constructions initially established in the Ulm groundplans go on to govern the elevations as well, even determining the location of details such as crockets and pinnacles. Hecht's purely arithmetical analyses reveal none of these important relationships. His stubborn rejection of geometrical hypotheses thus deprived his careful quantitative analyses of much of their value. In the end, therefore, his discussion of the Ulm drawings manages to be exhausting without being truly exhaustive.

Because Hecht's work appears to be so rigorous and comprehensive, at least on the surface, the publication of *Maß und Zahl in der gotischen Baukunst* has had a chilling effect on geometrical research in Gothic architecture, with particularly damaging consequences for the study of drawings. None of the major studies of Gothic drawings published over the last three decades has any substantial geometrical component.[39] Three

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[37] The groundplans of the parish church in Steyr that were drawn on both paper and parchment still have almost exactly the same scale, differing only by about .3 percent. See Johann Josef Böker, *Architektur der Gotik/Gothic Architecture* (Salzburg, 2005), p. 25.

[38] Hecht, *Maß und Zahl*, pp. 387–464.

[39] Geometrical and proportional issues are almost totally neglected in Peter Pause, *Gotische Architekturzeichnungen in Deutschland* (Bonn, 1973), and in Hans Koepf, *Die Gotischen Planrisse der Wiener Sammlungen* (Vienna, 1969). Bucher's *Architector* (1979) presents little in the way of specific geometrical analyses, although its introduction alludes to the importance of geometry. The comprehensive

important developments, however, have significantly advanced the state of geometrical research in Gothic architecture since the middle of the twentieth century. First, there has been a growing appreciation that Gothic design involved dynamic processes of geometrical unfolding, rather than just the rigid application of "*ad triangulum*" or "*ad quadratum*" design schemes.[40] Second, building surveys have grown more precise, allowing geometrical hypotheses to be tested far more rigorously than they could be in the decades around 1900.[41] Third, and perhaps most importantly, the emergence of affordable computers and computer-aided-design (CAD) software has begun to allow medievalists to process geometrical information with unprecedented flexibility and rigor. While these techniques have been profitably applied to the study of Gothic buildings, however, their potential for the examination of Gothic drawings has never been effectively exploited.[42] In this context, it makes good sense to revisit the geometry of Gothic architectural drawings, using the computer as an analytical tool.

A new Approach to the Geometry of Gothic Drawings

This book sets out to complement previous studies by providing this particular new perspective on Gothic architectural creativity. Its scope will be fairly broad, in chronological terms, since original design drawings survive from the thirteenth century to the end of Gothic era. In geographical terms, too, its total scope will be broad, since Gothic drawings survive from France, Spain, Italy, and the Holy Roman Empire. The

catalog edited by Roland Recht, *Les Bâtisseurs des cathédrales gothiques* (Strasbourg, 1989) does include geometric discussion, but not in the drawings section, and not related to the catalog of objects. Recht's *Le Dessin d'architecture* (Paris, 1995) says little of geometry. Böker's *Architektur der Gotik*, far and away the most ambitious publication ever undertaken on Gothic drawing, includes only a few short geometrical asides, although it does reinforce Hecht's valuable point about Gothic drawings being scaled.

[40] In the 1950s, already, there was Willy Weyres, "Das System des Kölner Chorgrundrisses," *Kölner Domblatt*, 16–17 (1959): 97–104. In the 1960s, Luc Mojon proposed a plausible design sequence for the choir of Bern Minster, in *Der Münsterbaumeister Matthäus Ensinger* (Bern, 1967), pp. 40–46. More recently, Stephen Murray and his students have proposed geometrical unfolding schemes for a variety of great churches in northern France. See, for example, Stephen Murray, *Notre-Dame, Cathedral of Amiens* (Cambridge, 1996). See also Paul von Naredi-Rainer, *Architektur und Harmonie: Zahl, Maß und Proportion in der abendländischen Baukunst* (Cologne, 1995).

[41] The emergence of laser-based surveying systems has been especially important in this connection. See *New Approaches to Medieval Architecture*, eds Robert Bork, William W. Clark, and Abby McGehee (Farnham, 2011). For application of these techniques to the study of vault design geometry, see David Wendland, "Cell Vaults: Research on Construction and Design Principles of a Unique Late-Mediaeval Vault Typology," in Karl Eugen Kurrer, Werner Lorenz, and Volker Wetzk (eds), *Proceedings of the Third International Congress on Construction History*, Brandenburg University of Technology, Cottbus, 20–24 May 2009 (3 vols, Berlin, 2009), pp. 1501–8.

[42] Many of the workshops charged with maintaining Gothic buildings now have sophisticated 3-D computer systems at their disposal. Meanwhile, in the academy, pioneering computer-based studies of buildings include Linda E. Neagley, "Elegant Simplicity: The Late Gothic Plan Design of St-Maclou in Rouen," *Art Bulletin*, 74 (1992): 423–40, and Michael T. Davis and Linda Elaine Neagley, "Mechanics and Meaning: Plan Design at Saint-Urbain, Troyes and Saint-Ouen, Rouen," *Gesta*, 39 (2000): 159–80. Neagley has established an impressive track record of computer use for geometrical research on Gothic buildings, but her one computer-based analysis of a Gothic drawing proves less fully satisfying, for reasons explained in Chapter 5. See "A Late Gothic Architectural Drawing at the Cloisters," in Elizabeth Sears and Thelma K. Thomas (eds), *Reading Medieval Images: The Art Historian and the Object* (Ann Arbor, 2002), pp. 90–99.

emphasis of this book, however, will necessarily be on the Germanic world, home to the vast majority of the surviving Gothic drawings. Of the roughly 600 such drawings known in the world today, well over 480 are in territories of the former Empire. Indeed, 442 of them are in a single city, Vienna.[43] Even in Vienna, many important drawings appear to have been lost, since there are no surviving drawings to document major projects such as the completion of the local workshop's masterpiece, the south spire of the Stephansdom, which was the tallest masonry structure in Europe upon its completion in 1433. One must imagine, therefore, that the Stephansdom workshop must once have possessed at least 500 drawings, and there is no reason to doubt that other major Gothic workshops once had comparably many. The surviving sample of Gothic design drawings, therefore, represents only a tiny fraction of the total originally produced. Significantly, too, most of these surviving drawings depict churches designed in what might be called the grand manner, with complex carved stone components that follow in the tradition pioneered by the great cathedral lodges of France. Brick architecture, despite its importance in so many regions of Europe, left behind few documentary traces of this kind. Secular architecture, and the simplified church architecture of the mendicant orders, are also under-represented in the drawings. Even within the elite tradition of church architecture that predominates in the drawings, certain building components received particular emphasis: façades, towers, tabernacles, and vaults. This is no coincidence, for these were not only among the most prestigious and visible elements of ecclesiastical architecture, but also some of the most complex and challenging to design. Thus, while surviving drawings cannot by themselves provide a full overview of Gothic architectural culture, they can reveal a tremendous amount about the design problems that Gothic draftsmen themselves saw as most interesting and exciting. This book, similarly, offers something less than a comprehensive analysis of all Gothic drawings, but it will demonstrate the geometrical methods of Gothic design in a new way, using several dozen of the most impressive Gothic drawings as examples.

The principal purpose of this book is to show, in detailed step-by-step fashion, how Gothic draftsmen could create architectural schemes of great subtlety using only simple tools and basic geometrical operations. Because this topic deserves careful attention in itself, this study will not deal at length with some other issues that have previously loomed large in discussions of Gothic geometry and design practice. Discussions of symbolism and numerology, for example, will have no place here; such associations may well have been important to patrons and builders, but they likely had little bearing on the precise mechanics of the design process. In this book, moreover, the emphasis will be on the original conception of each drawing, rather than on the complex process by which the ideas in the drawings were translated into full-scale architectural form. Foot units and systems of measure, therefore, will not figure prominently in this study, even though this topic has generated a fairly substantial specialized literature of its own.[44] As noted above, many Gothic drawings have simple and direct scale relationships with the finished

[43] The vast majority of these, 428 drawings, are in the Kupferstichkabinett in the Akademie der bildende Künste, which will henceforth be called simply the Vienna Academy collection. Another 14 important drawings are in the collection of the Wien Museum Karlsplatz, formerly known as the Historisches Museum der Stadt Wien. See Böker, *Architektur der Gotik*, esp. pp. 16, 415.

[44] For introductions to these issues, see, for example, Kidson, "A Metrological Investigation," and Eric Fernie, "A Beginner's Guide to the Study of Architectural Proportions and Systems of Length," in Eric

buildings, but others do not, which suggests that several different modes of translation from drafting table to stoneyard may have been used with rough simultaneity. Intermediate stages in this translation process are traceable in the full-scale drawings scratched into the masonry fabric of several completed buildings. In such instances the detail forms initially scratched into the building often differ subtly from those finally executed in three dimensions. This fact clearly demonstrates that the design process continued to move forward even after most of the composition had been worked out at small scale in the draftsman's studio.[45] Full-scale drawings, therefore, are certainly important records of the planning process, but the focus here will remain on the small parchment and paper drawings that most directly record the early phases of this process.

Gothic draftsmen certainly used geometrical operations to help generate their designs, but these geometrical steps did not take place in a vacuum. Tradition, functional requirements, and educated guesses about structural stability all would have informed the design process, establishing the basic outlines of the architectural scheme in ways that geometry by itself never could. Most Gothic designers, therefore, probably had at least a rough idea in mind even before sitting down at the drafting table. Geometrical play with the compass and rule then served to sharpen the focus, by generating specific trial lines that could be accepted or rejected depending on their usefulness in the overall scheme. In a sense, therefore, a Gothic design can be seen as an architectural topiary, in which geometry provides the quasi-random growth factor, while artistic judgment guides the pruning process. This dialog between growth and pruning helps to explain the organic quality characteristic of Gothic design.

The investigative method employed in this book closely resembles the Gothic design process itself. Here, once again, basic geometrical operations have been used to generate trial lines. In this project, however, the importance of a line can be judged by how well it matches lines already drawn on the medieval parchment, rather than by how well it matches a vague phantom in the mind's eye. This distinction, of course, makes the investigative process less open-ended than the original design process, but the resonance between the two has great methodological importance. In order to generate plausible hypotheses for testing, the researcher has to empathize with the original designer, imagining how a given design can be brought forth step by step on an initially blank sheet. It may be effectively impossible for citizens of the modern world to put themselves into the overall mindset of medieval draftsmen, but, in the narrowly circumscribed sphere of architectural geometry, it is possible to make educated guesses about the thought processes behind a given design. It makes sense to imagine, for example, that Gothic draftsmen usually approached their projects in a straightforward manner, establishing basic lines such as buttress axes and pier centerlines before going on to elaborate the smaller elements of the composition. As the following chapters will demonstrate, geometrical operations undertaken in this

Fernie and Paul Crossley (eds), *Medieval Architecture and its Intellectual Context: Studies in Honour of Peter Kidson* (London, 1990), pp. 229–38.

[45] As at the cathedrals of Clermont-Ferrand and Aachen, for example. See Michael T. Davis, "On the Drawing Board: Plans of the Clermont Terrace," in Nancy Wu (ed.), *Ad Quadratum: The Practical Application of Geometry in Medieval Architecture* (Aldershot, 2002), pp. 183–204; Wolfgang Schöller, "Le Dessin d'architecture à l'époque gothique," in Recht, *Bâtisseurs*, pp. 226–35; and Ulrike Heckner and Hans-Dieter Heckes, "Die gotischen Ritzzeichnungen in der Chorhalle des Aachener Doms," in Ulrike Heckner and Gisbert Knopp (eds), *Die gotische Chorhalle des Aachener Doms* (Petersberg, 2002), pp. 339–61.

order can indeed generate even the details of many complex drawings from the Gothic era. And, while there was certainly no single formula governing all Gothic design, the close relationships between many of these drawings make it possible to apply lessons learned in analysis of one drawing to other similar cases. Like medieval apprentices, therefore, modern researchers can learn through practice to appreciate and even recreate the subtleties of the Gothic design tradition.

Creative empathy may be one of the keys to understanding Gothic design, but scholarship in this field also depends on the rigorous testing of geometrical hypotheses that use of the computer facilitates. For every drawing considered in this book, photographs or digital scans of the original document were imported into the Vectorworks CAD environment. These scanned images almost perfectly replicate the proportions of the original drawings, as on-site measurement of the original drawings confirms.[46] On-site investigation also helps to locate compass prick holes, uninked construction lines, and other subtle traces of the draftsman's labor that may not be readily visible in scanned reproductions. These traces help to identify the points and lines that were important to the original draftsman. Once these clues are taken into account, the computer can be used to draw trial lines and polygons on top of the scanned drawing. In all the graphics in this book, the geometries of these added lines are perfect, in the sense that the squares are square, the circles circular, the verticals vertical, and so forth. These figures, in other words, have never been adjusted or "fudged" to match the scanned drawing. The computer, moreover, treats these figures as assemblages of perfectly thin lines, so that the user never has to worry about the issue of line width.[47] The combination of computer use and careful on-site examination of drawings, therefore, minimizes the problems of imprecision and ambiguity that had troubled earlier generations of geometrically inclined medievalists. This method, in fact, allows modern researchers to test geometrical hypotheses with unprecedented rigor.

Because the analytical method described here is so powerful, the greatest challenges in this area of research may now involve presentation rather than substance. The best way to present the results of this geometrical research would be to redraw each of the drawings in question, step by step, using the same tools as the original draftsmen did. That approach, unfolding over time, would fully capture the dynamics of the design process. It is difficult to replicate this procedural quality in a book without using a prohibitive number of illustrations for each case study. In a book of reasonably wide scope like this one, therefore, the results of the creative process must be compressed into a small number of images, typically two or three per drawing. Each of these illustrations explicitly shows geometrical figures to make visible geometrical operations that the original draftsmen likely used in creating their drawings. The draftsmen themselves, however, would not have had to draw complete geometrical figures like these in order to establish the layout of their compositions. A designer wishing to establish points outside an already constructed square, for example, might use his compasses to unfold the diagonals of the square to its baseline, but he would have had no need to actually draw in the arcs describing the path of the compass. Indeed, he would have had good reason not to, since such visible arcs

[46] In a number of cases where the parchment pieces of the drawing were obviously misaligned with each other at some point after its creation, the CAD system was used to rotate the pieces into their correct vertical alignments. In no case, however, were the proportions of the pieces altered from the original scans.

[47] This quality of the computer models, unfortunately, does not translate onto the printed page, where all the lines in both the original drawing and the overlaid figures must appear as ink bands of finite width.

would have appeared intrusive and distracting in the final drawing. Gothic draftsmen took care to leave few extraneous lines and marks on their drawings, especially on the large and impressive elevation drawings that were likely used for public presentation.[48] This logic helps to explain the relative dearth of construction lines that Hecht used to argue against the geometrical quality of Gothic design. In their groundplans and detail studies, however, Gothic draftsmen left many such lines, and even the great elevation drawings often include subtle geometrical traces that Hecht overlooked. In the illustrations for this book, the geometrical figures are conspicuous presences instead of mere traces, because the point of these illustrations is to make visible the logic of the creative process, rather than to create a purely architectural final image. Because of the complexity of the drawings in question, many of these geometrical overlays become quite dense. To clarify the logic behind their creation, a detailed explanatory text becomes necessary.

The joy of geometrical discovery, unfortunately, translates only very imperfectly into prose. In this sense, too, the modern researcher confronts a problem similar to that faced by the original Gothic designers, whose handbooks and recipes proved so much less satisfying than the new treatises emerging from Renaissance Italy. Because Gothic architectural practice emphasized procedural rules rather than the shape of the finished product, the only way to fully appreciate a Gothic drawing or building is to trace the likely sequence of steps in its creation.[49] Descriptions of this process can be wearying, as the previously cited passage from Roriczer's booklet on pinnacle design demonstrates. It is difficult to imagine any other way, however, to convey the true nature of the process to an audience of readers. Even Roriczer's decision to label all the major points in his pinnacle design cannot be dismissed as unreasonably pedantic, since there is no other technical or formal vocabulary available to precisely describe all the points the designer actually needed to consider. In the following chapters, therefore, similar labeling systems will also be used, even though this risks making the book read more like a mathematics textbook than a typical work of art-historical scholarship. Paul Frankl once wrote that "Anyone for whom geometric descriptions are tedious or irksome is unsuited for the history of architecture," but that Olympian comment is surely unrealistic.[50] Because uninterrupted descriptions of Gothic geometry can become numbing even for the heartiest enthusiasts, more general interpretive discussions frame each of the case studies in the following chapters, calling attention to the main aspects of each analysis. These discussions should help to make the key themes of each chapter accessible to the general reader. This, at least, is the hope, since geometrical research has implications that may be of interest to medieval scholars and Gothic enthusiasts of all stripes.

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[48] In some instances, in fact, the draftsmen appear to have used protective screens to keep their drawings from being punctured at key points where a compass had to be used repeatedly. In the drawing known as Rahn Plan A, for example (Figure 2.51), a series of concentric arcs were carefully drawn, quite obviously with a compass, to describe the inner arch profiles of a flying buttress. There is, however, no hole or prick point at their geometrical center. This effect could have been achieved by temporarily attaching a small parchment patch atop the main drawing to shield the centerpoint during the arc construction process.

[49] It is effectively impossible to provide an adequate simple overview of this process, showing only the "major" steps, because the location of seemingly major elements often depends on the establishment of seemingly minor elements earlier in the process. This is especially the case in drawings of towers, where details in the lower stories often help to determine the proportions of the upper stories.

[50] Paul Frankl, *Principles of Architectural History* (Cambridge, Mass., 1968), p. xv.

The chapters in this book build in essentially chronological order, so as to provide a reasonably comprehensive overview of Gothic architectural drawing production. Chapter 1 traces the origins of Gothic drawing, with an emphasis on developments in thirteenth-century France. The central documents in this section will be the portfolio of Villard de Honnecourt and the two façade drawings preserved in the so-called Reims Palimpsest. Chapter 2 considers the refinement of architectural drawing practices in the Rhineland in the decades bracketing 1300, paying particular attention to innovations made in the Strasbourg, Cologne, and Freiburg workshops. Chapter 3 provides a brief interlude south of the Alps, exploring the intrusion of geometric design practices into Italy. Modular approaches to design have usually been seen as typical of Italy, but evidence from the cathedral workshops of Orvieto and Siena will show that dynamic geometry was also used, at least in these elite ecclesiastical projects, in the early fourteenth century. Chapter 4 discusses the heroic late Gothic tower projects of central Europe, which are better documented in drawings than are any other architectural projects of the era. Crucial examples here include the towers of Prague Cathedral, Vienna's Stephansdom, Ulm Minster, Frankfurt's Bartholomäuskirche, and the cathedrals of Regensburg and Strasbourg. To provide a somewhat broader perspective, Chapter 5 explores a variety of other late Gothic drawings, from all across the European continent, which depict building plans, vaults, tabernacles, and façades. Chapter 6, finally, explores the emergence of classical design in Renaissance Italy, and the collision between this newly fashionable mode and the established traditions of the Gothic world.

Together, these chapters will argue for seven principal theses. First, and most importantly, this book will demonstrate that Gothic draftsmen developed their designs using a dynamic approach to geometry, in which sequences of simple geometrical operations could be combined to produce final forms of great complexity. This view of Gothic design practice differs significantly from the three basic options that have previously been proposed: the networks of symbolic lines first postulated in the nineteenth century; the rather static imposition of rigid geometrical figures suggested by Velte; and the purely modular system proposed by Hecht. Second, and in a related vein, this book will reveal that Gothic architectural ornament and articulation often calls attention to the very points that played especially important roles in the design process—a finding that deserves particular emphasis because it shows that Gothic articulation patterns are actually more logical and less capricious than most scholars have assumed. Third, this study will document remarkable continuity in northern Gothic design practice over the course of three centuries. The same geometrical relationships that govern the thirteenth-century drawings in Villard's portfolio can still be traced in the tower and vault designs from early sixteenth-century Germany and Austria, for example. Fourth, these case studies will show that Gothic designers employed only a small "tool-box" of basic geometrical operations, a fact which helps to make their work analytically tractable. Even relationships that have not received sustained scholarly attention can often be understood as permutations on simple principles. So, for example, the principle of polygon nesting seen in the rotated squares of Roriczer's "quadrature" (see Figure 0.2) can easily be generalized to the inscribing of other figures such as hexagons, octagons, and dodecagons within their generating circles. The relationship between octagons and their circumscribing circles, in particular, appears so frequently in Gothic drawings that "octature" deserves to be seen

alongside "quadrature" as one of the foremost proportioning strategies of the era (Figure 0.7). Fifth, the following chapters will demonstrate that Gothic draftsmen created their designs in a straightforward and practical manner, typically developing a single major geometrical composition on a single manageably sized parchment, even when designing tall towers and façades where many such sheets had to be assembled together to create the final image. This approach makes a great deal more sense, from the draftsman's point of view, than trying to establish global geometrical relationships over the whole composition, since tower drawings could reach heights of four meters and more. The parchments in these drawings, in fact, were often trimmed in ways that subtly express the geometrical armature underlying the architectural scheme. Sixth, this book will suggest that the progressive divergence of Italian from northern European design practice over the course of the Gothic era, and the divergence between Italian modular planning and northern geometrical planning in particular, helped to set the stage for the more obvious battle between Gothic and classical styles in the Renaissance. Finally, and in a related vein, the concluding chapter will argue that the triumph of the classical mode in the sixteenth century reflected not the inherent superiority of classical architecture, but rather the greater marketability of the classical mode, with its prestigious ancient associations and its easily legible formal order.

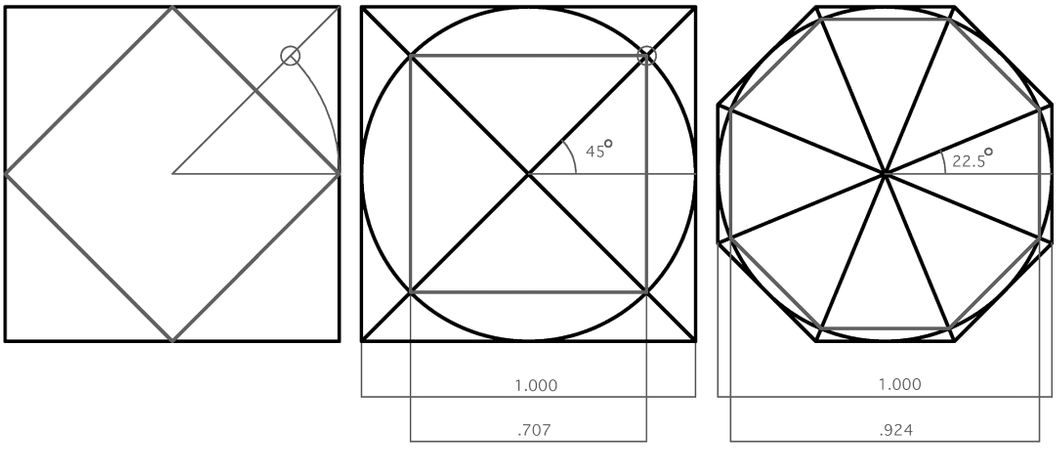

Figure 0.7 Basic proportioning figures of quadrature (at left and center) and octature (at right). In quadrature a rotated square can be inscribed directly within an original framing square, as at left. As the arc in this diagram indicates, the smaller square can be rotated back into alignment with the framing square, so that the center diagram results. This makes clear that the smaller square inscribes a circle that, in its turn, inscribes the framing square. The inscribed square is smaller than the framing square by a factor of √2. Its sides are thus .707 times as long as those of the framing square. In octature, at right, one considers a circle nested between two octagons instead of between two squares. The inner octagon is .924 times as large as the framing octagon. In trigonometric terms, .707=cos(45°) and .924=cos(22.5°).

Above and beyond arguing those specific points, this book aims to foster greater interest in the geometrical logic of the Gothic design process, a rich topic about which

much remains to be said. The drawings discussed here can certainly be studied further, and there are hundreds of other Gothic drawings that have never been analyzed from a geometrical perspective. The lessons of drawing-based research, moreover, have never been fully integrated with the results of recent geometrical and metrological research on full-scale Gothic buildings, and the field still suffers from the effects of Hecht's well-motivated but overly polemical critique. The scholarly enterprise, like the process of Gothic design, evolves through both growth and pruning, and geometrical research is an area now ripe for healthy future growth. If this book can help to re-establish geometrical study as a flourishing branch of Gothic architectural scholarship, it will have served its purpose.